Safety Verification for Power Systems using the Handelman Representation

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Background and motivation:
Verifying safety is as critical as stability synthesis for various nonlinear control applications, such as, electric power systems. We:
❖ Provide a barrier certificate term to obtain the region of safety (RoS)
❖ Find a positive definite polynomial as a barrier function using linear programming relaxation and Handelman representation for positive polynomials over polyhedral sets.

Technical approach:
- Keeping system trajectory in the safety limits is critical to avoid loss of generation and load in real world power system operations
- Encoding positivity of constructed barrier certificate in order to guarantee safety
- Linear programming relaxation by Handelman representation to prove the positivity of barrier function represented by polynomials on a compact complex polyhedral

Conclusion:
➢ Verifying safety is essential for systems that have safety critical limits, especially in systems with complex behaviors such as electric power systems.
➢ Utilizing Linear programming relaxation by Handelman representation to find the barrier certificate.
➢ Providing simulation results for two case studies that is validated by SoS programming.
➢ Considering extending the proposed method to find the region of safety in large-scale systems as a future work.
Consider a system governed by a set of differential equations as:
\[ \dot{x}(t) = f(x(t), d(t)), \] where, \( x(t) \in \mathbb{R}^n \) and \( d(t) \in \mathbb{R}^m \) is a large but bounded disturbance in a set \( D \):

**Safety:** Given \( (\dot{x}(t)), X_0, X_u, X \) and \( D \), the system has the safety property if there is no time instant \( T \geq 0 \) and no piecewise constant bounded disturbance \( d: [0, T] \to D \ni \varphi(t|x_0,d) \cap X_u \neq 0 \forall t \in [0, T] \)

**Region of Safety:** A set that only initializes trajectories with the property specified in Safety definition is called a region of safety. This is achieved thanks to a **barrier function** \( B(x) \):

\[ B(x): \mathbb{R}^n \to \mathbb{R} \] satisfies the following conditions using the Handelman representation:

\[ B(x) \leq 0 \quad \forall x \in X_I \; ; \quad B(x) > 0 \quad \forall x \in X_u \; ; \quad \frac{\partial B}{\partial x} f(x, d) \leq 0 \quad \forall (x, d) \in X \times D \]
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Handelman Representation: Let \( P = \{P_1, \ldots, P_m\} \) be polynomial bases, \( f_i = \{P_1^{n_1} \ldots P_m^{n_m} \mid n_j \leq D\} \) be the set of polynomials and \( K := \{x \in \mathbb{R}^n : P_j(x) > 0\} \) is A compact polyhedron if \( P_j(x) \) is \( a_jx - b_j \geq 0 \), then:

- If \( P(x) \) is strictly positive over a compact polyhedron \( K \), there exists a degree bound \( D > 0 \) such that:
  \[
P(x) = \sum_{f \in P} \lambda_i f_i \quad \forall \lambda_i \geq 0 \quad P(x) \text{ has a Handelman representation.}
  \]

Utilizing Linear programming relaxation by Handelman representation to find the barrier certificate for polynomial systems.

\[
\begin{align*}
\delta & = \begin{bmatrix} 0 \\ -20.44 \end{bmatrix} \omega \\
\omega & = \begin{bmatrix} 6.28 \\ -0.14 \end{bmatrix} \delta \\
\omega & = \begin{bmatrix} 0 \end{bmatrix} \omega \\
\end{align*}
\]

\[
X_u = \{[\delta \omega]^T : |\omega| \geq 0.5\}
\]
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