Robust Output Feedback Control Design for Inertia Emulation by Wind Turbine Generators

Samaneh Morovati, Graduate Student Member, IEEE, Yichen Zhang, Member, IEEE, Seddik M. Djouadi, Senior Member, IEEE, Kevin Tomsovic, Fellow, IEEE, Andrew Wintenberg, Graduate Student Member, IEEE and Mohammed Olama, Senior Member, IEEE

Abstract—Wind generation has gained widespread use as a renewable energy source. Most wind turbines and other renewables connected to the grid through converters result in a reduction in the natural inertial response to grid frequency changes. The doubly-fed induction generator (DFIG) can be controlled to compensate for this reduction and, in fact, provide faster response than traditional synchronous machines. This paper proposes to design observer based output feedback linear quadratic regulator (LQR) and \( H_{\infty} \) control laws to realize the inertia emulation function and deliver fast frequency support. The aim is to track the reference speed by a diesel synchronous generator (DSG) in order to reach the desired inertia. The control signal is computed based on a reduced order model using the balanced truncation technique. A comparison with selective modal analysis (SMA) and balanced truncation model reduction techniques is presented. Comprehensive results show the effective emulation of synthetic inertia by implementing the control laws on a nonlinear threephase diesel-wind system. The proposed technique is analyzed for different short circuit ratio (SCR) scenarios.

Index Terms—Inertia emulation, diesel-wind system, output feedback control, robust control, balanced truncation model reduction.

NOMENCLATURE

Mathematical Symbols

\[ \Delta \quad \text{Deviation from operating point} \]
\[ s \quad \text{Laplace operator} \]
\[ A, B, E \quad \text{State, control input, disturbance input matrices} \]
\[ C, D, F \quad \text{Output, control feed-forward, disturbance feed-forward matrices} \]

Physical Variables

All variables are in per unit unless specified.

\[ f_b \quad \text{Speed base of diesel generator [Hz]} \]
\[ H_D, H_w \quad \text{Diesel, wind turbine generator inertia constant [s]} \]

\[ H_{r,f} \quad \text{Reference model inertia constant [s]} \]
\[ i_{d,r,f}, i_{q,r,f} \quad \text{Instantaneous rotor current in } d, q \text{-axis} \]
\[ i_{d,s}, i_{q,s} \quad \text{Instantaneous stator current in } d, q \text{-axis} \]
\[ K_{P_r, K_I_r} \quad \text{Proportional, integral gain of torque controller} \]
\[ K_{P_q, K_I_q} \quad \text{Proportional, integral gain of reactive power controller} \]

*This research was sponsored in part by the Engineering Research Center Program of the National Science Foundation and the Department of Energy under NSF Award Number EEC-1041877 and the CURENT Industry Partnership Program, the National Science Foundation under NSF-ECCS-Awards 1711432 and 1509114, and a Joint Directed Research and Development (JDRD) grant.

S. Morovati, S. M. Djouadi and K. Tomsovic are with the Min H. Kao Department of Electrical Engineering and Computer Science, The University of Tennessee, Knoxville, TN 37996 USA (e-mail: smorovat@utk.edu; mdjouadi@utk.edu; tomsovic@utk.edu).

Y. Zhang is with Argonne National Laboratory, Lemont, IL 60439, USA (e-mail: yichen.zhang@anl.gov).

A. Wintenberg is with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109, USA (e-mail: awintenb@umich.edu).

M. Olama is with Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA (e-mail: olamahassem@ornl.gov).

I. INTRODUCTION

Diesel synchronous generators (DSGs) are a common choice for powering microgrids in remote locations. A renewable source can reduce the operating cost by partially replacing the usage rate of more expensive diesel generators [1]. Different types of renewable energy sources such as wind and solar are connected to the grid using power electronic interfaces that can ensure power injection at the rated grid frequency [2]. The variable nature of renewable power poses challenges for
frequency control in mixed diesel-renewable microgrids [3]. This variability can result in large frequency fluctuations without proper controls [4]. Furthermore, unacceptable frequency excursions caused by deterioration of inertial response in the presence of large disturbances can adversely impact system reliability [5]. To address the frequency stability challenges, renewable energy sources need to be equipped with innovative frequency control approaches that contribute to frequency regulation operations [6], [7]. Utilizing stored energy as synthetic inertial response, commonly referred to as inertia emulation, is one of the widely used approaches [8]. These controls can be employed either in grid connected mode or in island mode [9].

Although inertia emulation can be equipped for most converter-interfaced distributed energy resources (DERs), wind turbine generators (WTGs) are the most suitable candidate since the stored kinetic energy in the rotating mass can be readily utilized without additional storage. Existing inertia emulation methods generally couple the stored kinetic energy of WTGs to the rate of change of frequency (ROCOF) [10], [11]. The effective inertial response using such methods can be difficult to quantify as the emulated inertia constant is time varying [12], [13]. Therefore, the system performance cannot be guaranteed. The approach in [14] provides droop control for virtual synchronous generators as a specific control structure to estimate and control inertia. However, this approach needs to use the WTG as a voltage source and at the cost of a de-loaded operation. Besides, adequate frequency stability becomes critical with increasing renewable penetration. The main challenge from the control viewpoint is to keep the system frequency within specified bounds in the presence of disturbances.

In [1] and [15], the authors propose a novel inertia emulation controller for current-mode WTGs. The proposed controller employs the model reference control (MRC) paradigm to precisely emulate programmable inertia constant which guarantees performance on the frequency response. As we know, the MRC is more of a control task than a design method. The $H_\infty$-based state feedback control is employed in [1] and [15] to realize the MRC-based inertia emulation. In order to preserve the original states in the model reduction, the SMA-based model reduction method is used. State feedback control is not very practical since state measurements are not always available. The $H_\infty$ control usually leads to larger gains. Although a technical method is proposed in [1] and [15] to limit the size of the gains, the control signal still has a high peak, and the kinetic energy of WTGs is not optimally utilized.

To resolve the aforementioned issues, an output feedback LQR design is proposed in this paper for MRC realization. In this effort, a Luenberger observer is used to design dynamic output feedback LQR control laws. To simplify the control design, the balanced model reduction technique is used and compared with the SMA technique in capturing system characteristics. We verify the proposed method on a modified 33-node microgrid using a detailed full-order three-phase simulation model in Simulink. The proposed technique is analyzed for different SCR scenarios.

The significance of the proposed method in comparison to the other methods is two-fold. First, compared with the common inertia emulation methods for current-mode DERs that lead to time-varying synthetic inertia responses, we propose the MRC framework, which allows us to emulate the inertia precisely and provide guaranteed performance for the frequency response. We verify the proposed method on the modified 33-node microgrid using a detailed full-order three-phase simulation model in Simulink.

Second, since MRC is more of a control task than a design method, we propose the output feedback LQR and $H_\infty$ controls with the Luenberg observer to realize the MRC-based inertia emulation (IE). Compared with [1] and [15] which use state-feedback control design, the technical benefits are:

1) The output feedback design frees us from the SMA-based model reduction, which is necessary for state feedback control design since it keeps the original state information in the reduced-order model, and allows us to use a variety of model reduction techniques.
2) We employ balanced truncation model reduction and show that it provides higher accuracy compared to the SMA-based approach.
3) We employ both LQR and $H_\infty$ methods to design the output feedback control, and perform a thorough comparative study over these two methods as well as to the proportional-integrator based output feedback control.
4) Since state measurements are not available all the time, the output feedback control is more practical and can be readily applied in a practical implementation.

This paper is organized as follows. Section II briefly introduces the primary objective for inertial response. Section III presents the balanced truncation model reduction technique used for the diesel-wind system and a comparison with the SMA model reduction technique. Section IV describes the proposed observer-based LQR and $H_\infty$ controllers based on inertia emulation strategy. Simulation results are presented in section V, followed by the conclusion in section VI.

II. OBJECTIVE FOR INERTIAL RESPONSE

After a power disturbance, online synchronous generators will first limit the ROCOF by converting rotating kinetic energy into electric power, which is known as the inertial response. Then, as the rotor speed slows down the turbine-governor system senses speed deviations and acts to adjust the output of the prime movers to stabilize the rotor speed. The governor response is referred to as the primary frequency control [16]. The time scale of both responses are in terms of seconds. Due to the deadband and response time of the turbine-governor, the inertial response is dominant in the beginning period of frequency decline as shown in Fig. 1. The primary frequency response will then increase to regain power balance and stop the frequency decline. This process is governed by the swing equation:

$$2Hs\Delta \omega = \Delta P_m - \Delta P_d$$  \hspace{1cm} (1)

where $s$ is the Laplace operator, $2Hs\Delta \omega$ denotes the inertial response, $\Delta \omega$ denotes the primary frequency response and $\Delta P_d$ denotes the disturbance. With more renewable energy penetration, fewer synchronous generators will be committed.
leading to smaller inertia $H$ in the system, and potentially inadequate inertial response. Wind turbines, for example, are effectively decoupled from grid frequency and will not naturally respond to frequency changes. Thus, controls must be designed to limit the ROCOF if grid support is needed. Controlling the power output proportional and opposing the ROCOF is known as inertia emulation. The traditional approach of an inertia emulation strategy for a WTG is illustrated in Fig. 2. In a such strategy, the stored kinetic energy in a WTG will be released in proportion to ROCOF. The speed of the induction motion will decline due to the energy conversion of the WTG. The speed of the WTG is known as inertia emulation. The traditional approach of an inertia emulation strategy for a WTG is illustrated in Fig. 2, respectively, where $\Delta\omega$ with the support of WTG, that is, $\Delta\omega$ be the inertial response of the reference model $Hs\omega$. As described in [1] and [10], the configuration in Fig. 2 can only produce synthetic inertial response where the equivalent parameters are time varying and may be difficult to tune. This is easy to see if we rearrange (2) as follows:

$$2Hs\Delta\omega = \Delta P_m - \Delta P_d + G_w(s)K_ie\Delta\omega$$

This poses challenges for dynamic security assessment, stability analysis and system performance guarantees. See [1] and [12] for details on the derivation of equivalent parameters of frequency response model under emulated inertia.

$$2H - G_w(s)K_ie s \Delta\omega = \Delta P_m - \Delta P_d$$

Fig. 1. Typical frequency response after a generator trip.

Fig. 2. Traditional inertia emulation function within a wind turbine. (a) Detailed view. (b) Conceptual view.

Fig. 3. Model reference control-based inertial emulation control diagram.

The idea to achieve near-ideal synthetic inertial response of WTG can be recast as a tracking problem with respect to a dynamic reference model, known as the MRC. In the MRC, we define a frequency response model with desired parameters as the reference. The objective is to make the DSG speed precisely track the reference frequency using the support from the WTG as shown in Fig. 3. Intuitively, the frequency response of the augmented physical plant consisting of the diesel generator and the WTG will be the same as the reference of the response model. Therefore, the emulated inertia constant is close to the one of the reference model. To see this, let $2H_rf s\Delta\omega_f$ and $2H_D s\Delta\omega_d$ be the inertial response of the reference model and DSG in Fig. 3, respectively, where $H_rf$ is the desired inertia constant and $H_rf - H_D = H_ie > 0$. The power balance condition holds as:

$$\Delta P_d = 2H_rf s\Delta\omega_f = 2H_D s\Delta\omega_d + \Delta P_g$$

If the speed of DSG can track the speed of the reference model with the support of WTG, that is, $\Delta\omega_f \approx \Delta\omega_d$, then the following relation holds:

$$\Delta P_g \approx 2H_ie s\Delta\omega_d - 2H_D s\Delta\omega_d = 2H_ie s\Delta\omega$$

Therefore, the synthetic inertial response $2H_ie s\Delta\omega_d$ is emulated by the WTG. Traditional strategy in Fig. 2 can be considered as an open-loop control with respect to the WTG since no status information of the WTG is fed back to the
inertia emulation module. Since the MRC-based inertia emulation generates the control signal using both grid frequency and WTG states as shown in Fig. 3, it can compensate the negative effect induced by the motion dynamics of WTG. Nevertheless, the MRC is more of a control task than a design methodology. In the following, a mathematical model incorporating both the diesel and WTG will be derived, where an output feedback LQR and $H_{\infty}$ controllers will be designed to realize the MRC-based inertia emulation.

III. DIESEL-WIND SYSTEM MODELING

In this section, the dynamic model of the WTG is presented. The wind turbine model is assumed to be a type-3 WTG, which is one of the most common wind turbines used in practice. Type-3 wind turbines are also called DFIG-based wind turbines. Note that the proposed paradigm can be applied to any type of converter-interfaced DERs. But WTGs are more readily suitable due to their inherent kinetic energy.

A. Doubly-Fed Induction Generator and Converter Model

The converter of the wind turbine generator includes the rotor-side and grid-side converters, which control the speed of the generator and inject power into the grid, respectively [18]. Since, the rotor-side converter controls the generator speed by regulating the electromagnetic torque, the frequency support function should be included in this subsystem. The grid-side converter has less impact on the frequency support since the time scale of the DC regulation is much faster than the rotor-side control current loop for stability reasons [1].

The differential equations of the fluxes in the $dq$ axes and algebraic equations of the DFIG are given by:

$$\frac{d\lambda_{qs}}{dt} = \omega_b[V_{qs} - R_s i_{qs} - \omega_s \lambda_{ds}]$$  

(6)

$$\frac{d\lambda_{ds}}{dt} = \omega_b[V_{ds} - R_s i_{ds} + \omega_s \lambda_{qs}]$$  

(7)

$$\frac{d\lambda_{qr}}{dt} = \omega_b[V_{qr} - R_r i_{qr} - (\omega_s - \omega_r)\lambda_{dr}]$$  

(8)

$$\frac{d\lambda_{dr}}{dt} = \omega_b[V_{dr} - R_r i_{dr} + (\omega_s - \omega_r)\lambda_{qr}]$$  

(9)

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr}$$  

(10)

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr}$$  

(11)

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs}$$  

(12)

$$\lambda_{dr} = L_r i_{dr} + L_m i_{ds}$$  

(13)

The dynamics of the induction machine are presented in (14)-(19), where, $\tau_m$ and $\tau_e$ are the mechanical and electromagnetic torques ($\tau_e = (L_m/L_s)(\lambda_{qs} i_{dr} - \lambda_{ds} i_{qr})$). $\omega_{r,ref}$ is the filtered reference speed for the wind turbine generator and $\omega_{r,ref}$ is the reference rotor speed which is computed as an optimal speed based on the maximum power point tracking (MPPT) curve in relation with the measured electric power as shown in Fig. 4 (Eq. (20)) [19]. The state variables related to the speed controller of the WTG are represented as $x_1$ and $x_2$. Also, $x_3$ and $x_4$ are defined as state variables related to the reactive power controller. $Q_g$ and $P_g$ are the reactive and active power of the wind turbine generator [20].

$$\frac{d\omega_r}{dt} = \frac{\tau_m - \tau_e}{2H_w}$$  

(14)

$$\frac{d\omega_{r,ref}}{dt} = \omega_e(\omega_{r,ref} - \omega_{r,ref})$$  

(15)

$$\frac{dx_1}{dt} = K_{I_r}(\omega_{r,ref} - \omega + u_c)$$  

(16)

$$\frac{dx_2}{dt} = K_{I_q}(Q_{ref} - Q_g)$$  

(17)

$$\frac{dx_3}{dt} = K_{I_r}(i_{qr,ref} - i_{qr})$$  

(18)

$$\frac{dx_4}{dt} = K_{I_r}(i_{dr,ref} - i_{dr})$$  

(19)

$$\omega_{r,ref} = -0.67(P_g)^2 + 1.42(P_g) + 0.51$$  

(20)

The algebraic relations of the electric power are expressed in (21) and (22). The loop of algebraic equations is closed by the algebraic relations in (23) and (24) [1], where $\sigma = (L_r L_s - L_{qs}^2)/(L_r L_s)$ is the leakage coefficient of the induction machine.

$$P_g = V_{qs} i_{qs} + V_{ds} i_{ds} + V_{qr} i_{qr} + V_{dr} i_{dr}$$  

(21)

$$Q_g = V_{qs} i_{ds} - V_{ds} i_{qs} + V_{qr} i_{dr} - V_{dr} i_{qr}$$  

(22)

$$V_{qr} = x_3 + K_{P_r}(i_{qr,ref} - i_{qr})$$  

(23)

$$V_{dr} = x_4 + K_{P_r}(i_{dr,ref} - i_{dr}) - (\omega_s - \omega_r)(\sigma L_r i_{qr})$$  

(24)

where $i_{qr,ref}$ and $i_{dr,ref}$ are expressed by (25) and (26).

$$i_{qr,ref} = -\frac{L_s \tau_{r,ref}}{L_m \Psi_s}$$  

(25)

$$i_{dr,ref} = x_2 + K_{P_r}(Q_{ref} - Q_g)$$  

(26)

The model used for the DSG is the complete model as described in (27)-(29). This model shows speed changes of the diesel generator based on power, mechanical power and valve position variations [13], [16].

$$\frac{d\Delta \omega_d}{dt} = \frac{f_b}{2H_D} (\Delta P_m - (\Delta P_d - \Delta P_g))$$  

(27)

$$\frac{d\Delta P_m}{dt} = \frac{1}{\tau_d} (-\Delta P_m + \Delta P_v)$$  

(28)

$$\frac{d\Delta P_v}{dt} = \frac{1}{\tau_{mv}} (-\Delta P_v - (\Delta \omega_d/f_{base} R_D))$$  

(29)

Here, $\Delta P_d$ is the disturbance which is the measured power flow variation at a specified location [1] as shown in Fig. 3.
B. Model Reduction Technique

Model reduction is critical for control design of large-scale systems, such as the power grid, as they are governed by differential equations where the number of states can be extremely large [21]. The goal is to provide a low-dimensional model that has a similar response characteristics as the original system and allows a level of storage and computational requirements manageable for practical design and implementation [22].

The model reduction is also beneficial for implementation and deployment of dynamic feedback controllers which are dynamic systems and have the same order as the plant. The full-order plant contains faster electromagnetic dynamics and slower electromechanical dynamics. The former is less relevant to the frequency response, while the latter dominates the frequency behavior. Without model reduction, the controller dynamics will also contain the fast electromagnetic modes that are less relevant to the frequency response but require small steps to simulate, and consume considerable computation resources of the embedded system once deployed.

One popular model reduction technique is the balanced truncation, which is a simple efficient model reduction technique broadly used in reducing model orders of high order linear systems [23]. Balanced reduction was first introduced by Moore [24]. It has been shown to provide accurate reduced order model representations of state-space systems. Since the reduction procedure is based only on system inputs and outputs, model reduction may be heavily dependent on the scaling of the states. However, balanced truncation is independent of the particular system scaling since it uses balanced state space realizations [24].

This paper proposes the balanced reduction method for large scale power systems instead of the traditional reduction method defined as SMA [1]. Although the SMA method has a nice physical interpretation in many cases, it is not the ideal method from a control point of view, since it only relies on certain modes to reduce the order of a large model. We are suggesting a more accurate method that can maintain the main dynamical features of the whole system in the reduced model. The characteristic of this method can help us provide a reliable reduced order model and design a proper, optimized and robust controller to guarantee desired performance.

Assume a stable linear time-invariant system as illustrated by the n dimensional state-space model in (30).

\[
\dot{x}(t) = Ax(t) + Bu(t); \quad y(t) = Cx(t) \quad (30)
\]

In balanced truncation, a balanced realization is first obtained to make the controllability and observability Gramians \(Q_o\) and \(Q_o\) equal to the diagonal matrix of the Hankel singular values, i.e., \(\Sigma = \text{diag}(\sigma_1, \ldots, \sigma_N)\). These two Gramians should satisfy the Lyapunov equations:

\[
\begin{align*}
AQ_o + Q_oA^T + BB^T &= 0 \\
A^TQ_o + Q_oA + CT^TC &= 0
\end{align*}
\]

(31)

In addition, \(Q_c\) and \(Q_o\) form the bases for the controllable and observable subspaces [25]. Hence, the system is balanced when the controllability and observability Gramians are equal [26].

The controllability and observability Gramians are described as follows [26]:

\[
Q_o = \int_0^\infty e^{At} C^T Ce^{At} dt; \quad Q_c = \int_0^\infty e^{At} BB^T e^{A^Tt} dt
\]

(32)

In order to transform a realization into a balanced form, a coordinate transformation matrix \(T\) is needed to transform the balanced state vector \(x_b\) to the original state vector \(x\), where, \(x = Tx_b\), such that the transformed observability and controllability Gramians are diagonal and equal [24] as computed by the following equations:

\[
\dot{Q}_o = T^{-T}Q_oT^{-1}; \quad \dot{Q}_c = TQ_cT^T
\]

(33)

The transformation \(T\) can be computed by first calculating the matrix \(Q_{co} = Q_oQ_c\) [25] and determining its eigenmodes \(Q_{co} = T\Sigma^2T^{-1}\). Note that the transformation \(T\) is chosen such that the following identities are satisfied [26]:

\[
\dot{Q}_c = \dot{Q}_o = T^{-1}Q_oT^{-T} = T^TQ_oT := \Sigma
\]

(34)

So, the balanced state-space model (30) is obtained by taking \(TA_0T^{-1}A, C_0T^{-1}C\), and \(TBB = B\) [26]:

\[
\begin{align*}
\dot{x}_b(t) &= A_0x_b(t) + Bu(t); \quad y_b(t) = C_bx_b(t)
\end{align*}
\]

(35)

The balanced realization gives us the new order of states based on observability and controllability, where the first states are the most controllable and observable states [25]. Hence, (36) expresses the reduced order model by keeping \(n_r\) states \((x_1, \ldots, x_{n_r})\) that are the most controllable and observable states and most relevant from the control viewpoint [24].

\[
\begin{align*}
\dot{x}_r(t) &= A_rx_r(t) + B_ru(t) \\
y_r(t) &= C_rx_r(t)
\end{align*}
\]

(36)

Therefore, we can compute the reduced state space matrices using \(T_r = [I_r \ 0]T\) as:

\[
\begin{align*}
A_r &:= [I_r \ 0]T^{-1}AT \quad [I_r \ 0]; \quad B_r := [I_r \ 0]T^{-1}B \\
C_r &:= CT \quad [I_r \ 0]
\end{align*}
\]

(37)

The error bound of balanced truncation is given by [26]:

\[
\|y(t) - y_r(t)\|_2 \leq 2 \sum_{i=r+1}^{n} \sigma_i \|u(t)\|_2 \quad \forall u \in L^2
\]

(38)

where \(L^2\) denotes the space of finite energy signals (i.e., the measurable square integrable functions). In order to make the controller design procedure simple, a reduced linearized model about the equilibrium point for the type-3 WTG based on balanced reduction technique is used. A comparison with the SMA technique proposed in [1] is presented. It provides us with a benchmark on how close the reduced model is to the full order linearized model and its performance for all frequency ranges. The linearized full order model of the WTG around the equilibrium point is given as:

\[
\Delta \dot{x}_f = A_f \Delta x_f + B_fu; \quad \Delta y_f = C_f \Delta x_f + D_fu
\]

(39)
where 
\[
x_f = \begin{bmatrix} \lambda_{qs}, \lambda_{ds}, \lambda_{qr}, \lambda_{dr}, \omega_r, \omega_{f_{ref}}, x_1, x_2, x_3, x_4 \end{bmatrix}^T
\] (40)

The full order model is a 10th order model and \( \Delta \) gives the variation of each variable around the equilibrium. \( \Delta y_f \) is considered as the WTG power output variation, \( (P_g) \), due to the inertia emulation input. Then, the reduced order model of the WTG is expressed in (41), where we keep only the most controllable and observable states with the highest Hankel singular value magnitudes and truncate the rest of the state variables from the reduced realization. In other words, we are eliminating the states that are at the same time difficult to control and difficult to observe [22].

\[
\Delta \dot{x}_{red}(t) = A_{red}\Delta x_{red} + B_{red}u_c \\
\Delta y_{red} = C_{red}\Delta x_{red} + D_{red}u_c
\]
(41)

where, \( A_{red}, B_{red}, C_{red} \) and \( D_{red} \) are the state, control input, output and control feed-forward matrices of the reduced order model, respectively.

### IV. Control Design

In this section, two different control methods, a LQR and a static state feedback control for reference tracking are proposed. Since not all the state variables are available for measurements and only the reduced model is used in the control design stage, a Luenberger observer to estimate the measurements and only the reduced model is used in the control and difficult to observe [22].

To formulate the control problem, we consider the LQR cost function:

\[
J = \int_0^\infty [x^T Q x + u^T R u] dt
\]
(44)

where \( Q = C^T Q' C \) is a diagonal, symmetric, positive semi-definite matrix of \( \Delta \omega_d - \Delta \omega_{drf} \) weights and \( R \) is a diagonal, symmetric, positive definite matrix of control weights. The optimal control problem minimizes (44) over all controls \( u \in L^2(0, \infty) \) with the tracking constraint. The LQR problem has a unique solution for a controllable system and the optimal input \( u^* \) is given by [26]:

\[
u^* = -K x = -[K_p, K_{rf}] x
\]
(45)

Finally, the augmented closed loop system \( x_{aug} = [x_p, x_{rf}]^T \) is defined below:

\[
\dot{x}_{aug}(t) = \hat{A} x_{aug}(t) + \hat{B} x_{aug}(t) + \hat{E} d(t) \\
y(t) = y_p - y_{rf} = \hat{C} x_{aug}(t) + \hat{D} x_{aug}(t)
\]
(46)

where, \( d = [u_d, u_{df}] \), \( \hat{C} = [C_p, -C_{rf}] \), \( \hat{D} = [D_p K_p, D_p K_{rf}] \), \( \hat{A} = [\hat{A}_p 0; 0 \hat{A}_{rf}] \), \( \hat{B} = [B_p K_p; B_p K_{rf}] \) and \( \hat{E} = [E_p 0; 0 E_{rf}] \).

To compute the feedback law, the observer in (47) is used to estimate the states and we use the physical plant output measurements to get an output feedback controller as an inertia emulation controller. Therefore, the LQR based-observer controller is expressed as follows:

\[
\dot{x}(t) = \hat{A} \dot{x}(t) + \hat{B} u^* + L(y(t) - \hat{y}(t)) + \hat{E} d(t) \\
\hat{y}(t) = \hat{C} \dot{x}(t)
\]
(47)

which can be written as:

\[
\dot{x}(t) = (\hat{A} + \hat{B} K - L \hat{C}) \dot{x}(t) + L y(t) + \hat{E} d(t) \\
u_{ie} = K \dot{x}(t)
\]
(48)

where \( L \) is any matrix such that \( \hat{A} - L \hat{C} \) is stable [26].

The LQR feedback law is applied to the nonlinear system for reference tracking based on the \( H_\infty \) control structure fully...
The signal output feedback controller based on the observer expressed variables is designed based on the LQR or H∞ control techniques. Where the controller \( K \) is a symmetric linear matrix inequality (LMI), which can be computed based on [1]. Hence, similar to the LQR case, the use of the Luenberger observer gives a dynamic controller based on the computed static \( H_\infty \) control, and results in a dynamic output feedback controller.

The control structure for the LQR and \( H_\infty \) as output feedback control for reference tracking based on inertia emulation control is illustrated in Fig. 5. The controller \( \hat{K} \) is a dynamic output feedback controller based on the observer expressed in (48). The signal \( d \) represents disturbances and \( Z \) represents measurements while \( y \) denotes the observed outputs (\( \Delta \omega_d \) and \( \Delta \omega_{dr} \)) from the physical plant. The dynamic of the controller \( K \) can be represented by

\[
\begin{align*}
\dot{x}(t) &= (\hat{A} + \hat{B}K - L\hat{C})\dot{x}(t) + Ly(t) \\
u_{ic}(t) &= Kx(t)
\end{align*}
\]

where the controller \( K \) is designed based on the LQR or \( H_\infty \) control techniques.

A comparison of the accuracy using balanced truncation and SMA methods are presented in Fig. 6. As shown, we can capture the full order model precisely. In addition, \( H_\infty \) norms for the difference between the reduced model transfer function and the full order transfer function (\( ||G_f - G_r||_\infty \)), where \( ||\cdot||_\infty \) is the \( H_\infty \)-norm, are given in Table I. Balanced truncation can reduce the order of the model with a much lower \( H_\infty \) norm, validating the accuracy.

### V. Numerical Results

The proposed controllers are applied to a modified 33-bus microgrid simulated using MATLAB Simulink platform [27], [28]. The closed-loop system performance is tested using the single diesel-wind system described in [1]. The WTG model is modified based on the DFIG in the Simulink demo library by changing the aerodynamic model to the one detailed in [20], where a two-mass model is reduced to the swing equations with combined inertia of the turbine and generator [1].

For simulation purposes, time constants of turbine-governor system in the reference model are considered equal to that in the diesel synchronous generator. Moreover, we only consider tuning the inertia constants of the reference model and do not emulate load damping effects [29]. The system parameters are given in Appendix A.

#### A. Model Reduction Results

The reduced 4th order model of the WTG is expressed in (53). Since there are only 4 states with the highest Hankel singular value magnitudes, we keep only the 4 most controllable and observable states and truncate the rest of the state variables from the reduced realization. In other words, we are eliminating the states that are at the same time difficult to control and difficult to observe [22]. The Hankel singular values are:

\[
H_s = \begin{bmatrix} 1.6 & 1.2 & 1.1 & 1.1 & 0.02 & 0.01 & 0.003 \\
0.005 & 0 & 0 \end{bmatrix}^T
\]

Observe the sharp decrease in the magnitudes of the singular values after the 4th one justifying keeping only 4 states variables and eliminating the rest in the following reduced model:

\[
\begin{align*}
\Delta \dot{x}_{red}(t) &= A_{red}\Delta x_{red} + B_{red}u_c \\
\Delta y_{red} &= C_{red}\Delta x_{red} + D_{red}u_c
\end{align*}
\]

where,

\[
A_{red} = \begin{bmatrix}
-638.5 & -35.1 & 72.05 & -288.3 \\
35.1 & -0.06 & 0.13 & -101.1 \\
-72.05 & 0.13 & -0.28 & 353.7 \\
-288.3 & 101.1 & -353.7 & -135.6
\end{bmatrix},
\]

\[
B_{red} = \begin{bmatrix}
-45.87 \\
0.37 \\
-0.8 \\
-17.25
\end{bmatrix},
\]

\[
C_{red} = \begin{bmatrix}
45.87 & 0.37 & -0.8 & 17.25
\end{bmatrix}^T,
\]

\[
D_{red} = 0.94
\]

A comparison of the accuracy using balanced truncation and SMA methods are presented in Fig. 6. As shown, we can capture the full order model precisely. In addition, \( H_\infty \) norms for the difference between the reduced model transfer function and the full order transfer function (\( ||G_f - G_r||_\infty \)), where \( ||\cdot||_\infty \) is the \( H_\infty \)-norm, are given in Table I. Balanced truncation can reduce the order of the model with a much lower \( H_\infty \) norm, validating the accuracy.

---

**Fig. 5.** Output feedback observer-based control.

**Fig. 6.** Singular values of full order model and reduced models.
### Table I

<table>
<thead>
<tr>
<th>Reduction Method</th>
<th>$H_\infty$ Norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced Truncation</td>
<td>0.065</td>
</tr>
<tr>
<td>SMA</td>
<td>3.707</td>
</tr>
</tbody>
</table>

The responses of the nonlinear system and the reduced order linear model are compared in Fig. 7 in order to validate the latter under a step signal input. As shown in Fig. 7, the modeling error using the balanced truncation is not significant. The mean squared error between the reduced order model and nonlinear model for the speed of the DSG ($\omega_D$), WTG active power variation ($P_{\text{act}}$), and mechanical power variation of diesel generator ($P_{\text{mech}}$), captured as $0.034\text{Hz}$, $1.4 \times 10^{-6}\text{W}$ and $0.0013\text{MW}$, respectively. The closed-loop system performance subject to a step load change at bus 18 as a disturbance is considered for two different cases.

### B. Case I

For the first case, the desired inertia in the reference model is two seconds, that is, $H_{r,f} = 2$, and the inertia constant of the DSG is set to one second, that is, $H_D = 1$. In other words, we will emulate one second inertia constant from the WTG, that is, $H_{r,e} = 1$. Under the MRC paradigm, we compare three design approaches, that is, LQR, $H_\infty$, and a simple proportional–integral (PI) controller. We also compare the MRC paradigm with the conventional inertia emulation method. The ROCOF is obtained by a washout filter $k_i s / (1 + 0.01 s)$. We index the aforementioned controllers as follows:

- Controller 1: MRC-based IE with LRQ realization
- Controller 2: MRC-based IE with $H_\infty$ realization
- Controller 3: MRC-based IE with PI realization
- Controller 4: Conventional IE using a washout filter

The feedback gains obtained for Controllers 1 and 2 are given in (54) and (55):

$$K_{iqr} = \begin{bmatrix} -9.98 & -2.27 & 0.01 & -4.25 & 0.04 & -0.41 \\ -0.47 & 9.94 & 1.06 & -0.04 \end{bmatrix}$$  \hspace{1cm} (54)

$$K_{H_\infty} = \begin{bmatrix} 83.01 & 2.98 & 0.17 & 18.74 & 0.42 & 2.33 & 2.24 \\ -83.38 & -2.39 & 0.12 \end{bmatrix}$$  \hspace{1cm} (55)

The PI gain for Controller 3 is obtained via the pidtune function in Matlab and given as $K_p = 0.0113$, $K_I = 0.6471$. The IE gain for Controller 4 can be determined based on Eq. (3), that is, $k_i = -H_{r,e}/(2f_b)$. To emulate one second inertia constant, we set $k_i = -0.03$. The closed-loop performance is illustrated in Fig. 8. As shown, the synthetic inertia constant is accurately emulated using Controllers 1 and 2. However, both Controllers 3 and 4 have tracking errors. Fig. 8 (a) and (b) present the control inputs and the power outputs of the WTG, respectively. Note that there exists a weak inertial response for the field-oriented controlled DFIG-based WTG even without a supportive controller, and this response is sensitive to the rotor current-controller bandwidth and cannot provide the exact synthetic inertia. To have a precise comparison, the tracking error for each realization is shown in Fig. 8 (d). It can be readily observed that Controller 2 outperforms all controllers followed by Controller 1 with the objective to remove tracking error to get precise emulated inertia.

### C. Case II

In the second case, setting the desired inertia to five seconds ($H_{r,f} = 5$), the closed-loop performance using MRC based IE with different realizations is illustrated in Fig. 9. We use the same indices as in the previous subsection to denote the controllers. In this case the computed feedback laws (56) and (57) for Controller 1 and 2 are

$$K_{iqr} = \begin{bmatrix} -9.98 & -2.27 & 0.01 & -4.25 & 0.04 & -0.41 & -0.47 & 9.87 & 0.33 & -0.05 \end{bmatrix}$$  \hspace{1cm} (56)

$$K_{H_\infty} = \begin{bmatrix} 288.9 & 5.86 & 0.20 & 44.43 & 1.35 & -3.51 & 13.30 \\ -289.16 & -1.21 & 0.019 \end{bmatrix}$$  \hspace{1cm} (57)

The re-tuned PI gains for Controller 3 are $K_p = 1.29$, $K_I = 4.56$. For conventional IE (Controller 4), based on $k_i = -H_{r,e}/(2f_b)$, to emulate four seconds inertia constant, we set $k_i = -0.12$. The performance of all controllers is shown in Fig. 9. As seen in the figure, small tracking errors are obtained by Controllers 1 and 2, that is, the closed-loop system can emulate the desired inertia under the presence of disturbance. It is clear that if the desired inertia defined by the reference model increases, Controllers 3 and 4 are unable to track the reference frequency well and their tracking errors are higher relative to the LQR and $H_\infty$ controllers. It can also be observed that Controller 2 outperforms all controllers followed by Controller 1. For Cases I and II, the $H_\infty$ controller performs better than the LQR and PI controllers since it has improved robustness properties in the presence of plant uncertainties, that is the discrepancy between the reduced order linear model and the full order nonlinear model.

### D. Control Performance for Different Short Circuit Ratios

As known, the SCR is often used as an index for the connection strength. The SCR of a strong grid is discussed in [30], [31], [32]. The SCR is defined as the ratio between short circuit apparent power from a 3-line to ground fault at a given

---

Fig. 7. Response comparison of nonlinear and reduced order model physical plant. (a) Step input. (b) WTG active power variation. (c) Speed of DSG. (d) Mechanical power variation of diesel generator.
Fig. 8. Closed-loop performance under MRC based IE with LQR, $H_\infty$, PI controllers realization and conventional IE with desired inertia set to two second. (a) Control input. (b) Active power variation of WTG. (c) Speed of DSG. (d) Tracking error.

Fig. 9. Closed-loop performance under MRC based IE with LQR, $H_\infty$, PI controllers realization and conventional IE with desired inertia set to five second. (a) Control input. (b) Active power variation of WTG. (c) Speed of DSG. (d) Tracking error.

The performance for the MRC based inertia emulation with LQR and $H_\infty$ controllers, by setting all parameters and controllers similar to Case II, are provided in Fig. 10 and Fig. 11, respectively. As it is clear, both proposed controllers can emulate the desired inertia with a small tracking error. The tracking error varies in a negligible range for all scenarios. The captured SCR lower bound by simulations is 0.26, that is the system performance with the proposed techniques is guaranteed for SCR $\geq 0.26$.

Fig. 10. Closed-loop performance under MRC based IE with LQR realization for different SCRs. (a) Control input. (b) Active power variation of WTG. (c) Speed of DSG. (d) Tracking error.

Fig. 11. Closed-loop performance under MRC based IE with $H_\infty$ realization for different SCRs. (a) Control input. (b) Active power variation of WTG. (c) Speed of DSG. (d) Tracking error.

location in the power system to the rating of the inverter-based resource connected to that location [33]. As the numerator of SCR relies on the specific measurement location, this location is usually stated along with the SCR number that is defined as:

$$SCR = \frac{MVASC}{MW_n} \quad (58)$$

where MVASC is the short circuit MVA level at the point of interconnection (POI) without the current contribution of the WTG, and MWn is the nominal power rating of the WTG being connected at the POI. Here to analyze the sensitivity of the proposed technique, the closed-loop system performance for different SCR values is provided by implementing the MRC based inertia emulation with LQR and $H_\infty$ controllers. The SCR values for three different scenarios in a range of (1.95,5) are provided in Table II where the MWn = 1.1MVA.

<table>
<thead>
<tr>
<th>MVASC</th>
<th>SCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>1.95</td>
</tr>
<tr>
<td>3.4</td>
<td>3.1</td>
</tr>
<tr>
<td>5.6</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Table II
SCR VALUE FOR DIFFERENT SCENARIOS
VI. CONCLUSIONS

In this paper, new output feedback LQR and $H_{\infty}$ control laws for inertia emulation using balanced truncation and the Luenberger observer are proposed. The controllers are applied to the full order nonlinear model and compared favorably to a PI controller and a conventional inertia emulation using a washout filter. The diesel generator speed follows the reference model in the time scale of inertial response, and accurate emulated inertia is guaranteed by generating additional active power from the WTG. The performance of the closed loop system shows improved accuracy with the $H_{\infty}$ controller relative to the LQR controller, although they both achieve the desired frequency response. Therefore, without providing a specified margin for the frequency, adequate frequency response with robustness in the presence of disturbances can be achieved by setting the desired inertia based on the network operating point. The proposed technique is analyzed for different SCR scenarios where a lower bound to guarantee the performance is obtained.

APPENDIX A

Variables are in per unit unless specified otherwise. $S_{base} = 1.1$ MVA, $V_{base} = 575$ V, $f = 377$ (rad/s).

Operating condition: Wind speed: $11$ m/s, $P_g = 0.8$, $Q_g = 0$, $V_{ds} = 0$, $V_{qs} = 1$.

Equilibrium point for the linearization: (for the dynamic equations) $\lambda_{ds} = 1.015$, $\lambda_{qs} = 0.002$, $\lambda_{dr} = 1.041$, $\lambda_{qr} = 0.223$, $\omega_r = 1.19$, $x_1 = -0.641$, $x_2 = 0.261$, $x_3 = 0.011$, $x_4 = 0.005$.

(For the algebraic equations) $i_{ds} = 0.084$, $i_{qs} = -0.631$, $i_{dr} = 0.261$, $i_{qr} = 0.671$, $V_{dr} = -0.196$, $V_{qr} = 0.484$.

Diesel generator: rated power: $1$ MW, $H_D = 1$ (s), $\tau_{sm} = 0.1$ (s), $\tau_D = 0.2$ (s).

Wind turbine generator: rated power: $1$ MW, $H_w = 2$ (s), $\tau_{sm} = 0.1$ (s), $\tau_T = 0.2$ (s), $K_{Ir} = 0.1$, $K_{Pr} = 0.6$, $K_{Iq} = 8$, $K_{Pq} = 5$, $K_{Pq} = 1$.

REFERENCES


Samaneh Morovati (GS’16) received the B.S. degree in Electrical Engineering with a concentration in control systems from Imam Khomeini International University, Iran, in 2011 and the M.S. degree in Electrical Engineering with a concentration in control and signalling from Iran University of Science and Technology, Iran, in 2017. She is currently pursuing the Ph.D. in Electrical Engineering at the University of Tennessee, Knoxville, TN, USA. Her main research interests include control system theory, distributed and decentralized control, power system dynamics, renewable energy and smart grids.

Yiehen Zhang (’13–’M’18) received the B.S. degree in electrical engineering from Northwestern Polytechnical University, Xi’an, China, in 2010, the M.S. degree in electrical engineering from Xi’an Jiaotong University, Xi’an, China, in 2012, and the Ph.D. degree in electrical engineering from the Department of Electrical Engineering and Computer Science, University of Tennessee, Knoxville, TN, USA, in 2018. He was also a research assistant with the Oak Ridge National Laboratory from 2016-2018. He is currently a post-doctoral appointee with the Energy Systems Division, Argonne National Laboratory. His research interests include filtering and control of systems under communication constraints, modeling and control of wireless networks, control systems and applications to autonomous sensor platforms, electromechanical and multi-bile communication systems, in particular smart grid and power systems, control systems through communication links, networked control systems, and model reduction for aerodynamic feedback flow control.

Seddik M. Djouadi (M’99-SM’20) received the Ph.D. degree from McGill University, the M.Sc. degree from University of Montreal, both in Montreal, the B.S. (with first class honors) from Ecole Nationale Polytechnique, Algiers, El Harrach, Algeria, all in electrical engineering in 1999, 1992, and 1989, respectively. He is currently a Full Professor in the Electrical Engineering and Computer Science Department, University of Tennessee, Knoxville. Prior to joining the University of Tennessee, he was an Assistant Professor in the University of Arkansas at Little Rock, and held post-doctoral positions in the Air Force Research Laboratory and Georgia Institute of Technology, where he was also a Design Engineer with American Flywheel Systems Inc. His research interests include filtering and control of systems under communication constraints, modeling and control of wireless networks, control systems and applications to autonomous sensor platforms, electromechanical and multi-bile communication systems, in particular smart grid and power systems, control systems through communication links, networked control systems, and model reduction for aerodynamic feedback flow control.

Dr. Djouadi received five US Air Force Summer Faculty Fellowships, and five Oak Ridge National Laboratory Summer Fellowships. He received the Best Paper Award in the 1st Conference on Intelligent Systems and Automation 2008, the Ralph E. Powe Junior Faculty Enhancement Award in 2005, the Tibbet Award with AFS, Inc., in 1999, and the American Control Conference Best Student Paper Certificate (best five in competition) in 1998. He was selected by Automatica twice as an outstanding reviewer for 2002–2003 and 2003–2004.

Kevin Tomsovic (F’07) received the BS from Michigan Tech. University, Houghton, in 1982, and the MS and Ph.D. degrees from University of Washington, Seattle, in 1984 and 1987, respectively, all in Electrical Engineering. He is currently the CTO Professor with the Department of Electrical Engineering and Computer Science, University of Tennessee, Knoxville, TN, USA, where he also directs the NSF/DOE ERC, the Center for Ultra-Wide-Area Resilient Electric Energy Transmission Networks (CURENT), and has also served as the Electrical Engineering and Computer Science Department Head from 2008 to 2013. He was on the faculty of Washington State University, Pullman, WA, USA, from 1992 to 2008. He held the Advanced Technology for Electrical Energy Chair with Kumamoto University, Kumamoto, Japan, from 1999 to 2000, and was the NSF Program Director with the Electrical and Communications Systems Division of the Engineering Directorate from 2004 to 2006. He has also held positions at National Cheng Kung University and National Sun Yat Sen University in Taiwan from 1988-1991 and the Royal Institute of Technology in Sweden from 1991-1992. He is a Fellow of the IEEE.

Andrew Wintenberg (GS’20) was born in Knoxville, Tennessee, USA, in 1996. He received his B.S. degree in Electrical Engineering and Mathematics from The University of Tennessee, Knoxville, USA, in 2018 and his M.S. degree in Electrical Engineering from the University of Michigan, Ann Arbor, in 2020, where he is currently a Ph.D candidate. His research interests include the safety and security of discrete-event and cyber-physical systems through formal methods and abstraction.

Mohammed Olama (S’98-M’08-SM’19) received the B.S. and M.S. (Hons.) degrees in electrical engineering from the University of Jordan, Amman, Jordan, in 1998 and 2001, respectively, and the Ph.D. degree from the University of Tennessee, Knoxville, TN, USA, in 2007. He is currently a Senior Research Scientist with the Computational Sciences and Engineering Division, Oak Ridge National Laboratory, Oak Ridge, TN, USA. He is also an Adjunct Associate Professor with the EECS Department at the University of Tennessee, Knoxville, TN, USA.

He has more than 160 archival publications (including journals, conference proceedings, book chapters, and technical reports) in addition to numerous presentations at professional conferences and international symposia. His research interests include smart grid and smart buildings; smart grid communications and control; building-to-grid integration; cyber-physical systems; complex systems; wireless communications; healthcare data analytics; machine learning; artificial intelligence; statistical signal processing; and discrete-event simulation.