Semi-Analytical Electromagnetic Transient Simulation Using Differential Transformation

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# Introduction

<table>
<thead>
<tr>
<th></th>
<th>Transient stability simulation</th>
<th>Electromagnetic transient simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time step</strong></td>
<td>Milliseconds</td>
<td>Microseconds</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td>Positive-sequence, three-phase balanced model</td>
<td>Detailed three phase model</td>
</tr>
<tr>
<td><strong>Voltages and currents</strong></td>
<td>Phasor value</td>
<td>Instantaneous value</td>
</tr>
<tr>
<td><strong>Dynamics of interest</strong></td>
<td>Slower dynamics, e.g., electromechanical interactions among generators</td>
<td>Fast dynamics, e.g., electromagnetic interactions among power electronics devices</td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td>Power frequency</td>
<td>Wide range of frequency</td>
</tr>
<tr>
<td><strong>Advantages</strong></td>
<td>Fast and efficient</td>
<td>Capture detailed fast dynamics, simulate unbalanced faults</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td>Suitable only for low frequency dynamics</td>
<td>High computation cost</td>
</tr>
</tbody>
</table>

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State-space EMT simulation

State-space method

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]  \hspace{1cm} (1)

\( x \) is the state variables vector, \( u \) is the system input vector, \( y \) is the system output vector.

Example

\[ \begin{bmatrix} i \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} v_s \]  \hspace{1cm} (2)

\( v_{out} = -Ri + v_s \)

Fig.1. An RLC circuit with an ideal voltage source

Work in this paper:

Based on state-space-represented EMT simulations, this paper proposes a semi-analytical-solution method that repeatedly utilizes a high-order approximate solution of the EMT equations at longer time steps.

Differential transformation method

Considering a continuous function \( f(t) \), the \( k \)th order differential transform (DT) of \( f(t) \) is defined by

\[
F(k) = \frac{1}{k!} \left[ \frac{d^k f(t)}{dt^k} \right]_{t=t_0}
\]  

(3)

Also, from the Taylor series

\[
f(t) = \sum_{k=0}^{\infty} F(k) (t - t_0)^k
\]  

(4)

Replace all the variables in equations by their Taylor series composed of DT and equal the like terms of \( (t-t_0)^k \) in the equation, then linear equations of different order DTs can be established.

Original continuous functions \( k \)th order DT

\[
\begin{align*}
f(t) & = c \\
f(t) & = cg(t) \\
f(t) & = g(t) \pm h(t)
\end{align*}
\]

\[
\begin{align*}
F(k) & = c\eta(k), \quad \eta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \\
F(k) & = cG(k) \\
F(k) & = G(k) \pm H(k)
\end{align*}
\]

\[
\begin{align*}
f(t) & = \frac{dg(t)}{dt} \\
f(t) & = \sin(h(t)) \\
g(t) & = \cos(h(t))
\end{align*}
\]

Finally, any state variables in the equation can be approximated by their DTs up to the \( k \)th order, e.g.

\[
f(t) \approx \sum_{k=0}^{i} F(k) (t - t_0)^k
\]  

(5)

Case study

Steady state: 0-1 s
During fault: Bus 7 grounded for 5 cycles
Post fault: Fault cleared

Post fault simulation lasting for 2 seconds
20th-order DT method using a time step of 100 μs is compared with the benchmark

Fig. 2. A two-area system

Fig. 3. Synchronous generator relative rotor angle
Fig. 4. Synchronous generator frequency deviation
Fig. 5. Three-phase terminal current of generator #2
Fig. 6. Three-phase voltage at bus #7
## Case study

(1) **Comparison between the differential transformation with different orders**

<table>
<thead>
<tr>
<th>Error (p.u.)</th>
<th>DT (10th order)</th>
<th>DT (20th order)</th>
<th>DT (30th order)</th>
<th>DT (40th order)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time step (μs)</td>
<td>Time cost (s)</td>
<td>Time step (μs)</td>
<td>Time cost (s)</td>
</tr>
<tr>
<td>10^-2</td>
<td>72</td>
<td>81</td>
<td>200</td>
<td>58</td>
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<tr>
<td></td>
<td>330</td>
<td>53</td>
<td>412</td>
<td>58</td>
</tr>
<tr>
<td>10^-3</td>
<td>58</td>
<td>100</td>
<td>180</td>
<td>64</td>
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<tr>
<td></td>
<td>306</td>
<td>57</td>
<td>382</td>
<td>62</td>
</tr>
<tr>
<td>10^-4</td>
<td>46</td>
<td>1162</td>
<td>158</td>
<td>73</td>
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<tr>
<td></td>
<td>284</td>
<td>62</td>
<td>365</td>
<td>68</td>
</tr>
</tbody>
</table>

(2) **Comparison between the 30th order differential transformation with conventional numerical methods**

<table>
<thead>
<tr>
<th>Error (p.u.)</th>
<th>Modified Euler</th>
<th>4th order Runge Kutta</th>
<th>DT (30th order)</th>
<th>Trapezoidal-rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time step (μs)</td>
<td>Time cost (s)</td>
<td>Time step (μs)</td>
<td>Time cost (s)</td>
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<td>10</td>
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<tr>
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<td>2100</td>
<td>5</td>
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<td>230</td>
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<tr>
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<tr>
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<td></td>
<td></td>
<td>400</td>
<td>600</td>
</tr>
</tbody>
</table>

(1) High order DT could enlarge the time step.

(2) When the order is too high, the burden of computing high order terms decreases the overall efficiency.

(1) The DT based approach enables large time steps and thus reduces time costs.

(2) Trapezoidal-rule method is A-stable and enables large time step, but iterations are required to achieve high accuracy, and increases time cost.
Acknowledgements

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