Computationally Robust Line Outage Detection and Identification in Three-Phase Networks

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Abstract—**Detection and identification of individual phase outages remains a challenging problem due to insufficient metering in three-phase unbalanced power networks. This problem was tackled for the transmission systems in our previous work. In this paper, this work is extended to detect phase outages in three-phase unbalanced systems using only the sparse estimation method. In addition, further improvements are introduced to increase the estimation accuracy for the virtual power injections at the terminal buses of disconnected lines, once the disconnected line is identified by sparse estimation methods. Simulation results are provided to experimentally validate the increased accuracy in detecting phase outages while decreasing the computational time by using the proposed approach.**

Index Terms—**Sparse Estimation, Line Outage, Phase Outage, LASSO, Outage Detection, Distribution System Line Outage, Ordinary Least Square**

I. Introduction

Monitoring the real-time system state and network model by the system operators is crucial for them to take timely preventive actions under extreme conditions. The commonly used monitoring application remains to be the state estimator. However, almost all state estimators implicitly assume a perfectly known network model and attribute any discrepancies to measurement errors. Thus, in case of an unreported outage of an individual phase or the entire line, estimated state will be significantly biased impacting all other applications that rely on state estimator's results. Therefore, having a standalone tool for detecting the line outages will help to increase the overall system reliability. Historical examples such as the 2003 Northeast blackout [1] when the state estimator failed to provide proper system states during unreported line outages constitute the main motivation for this work. Relying on the state estimator results without the correct network model may result in partial or full network blackout as described in [2], [3].

In recent years, various alternative solutions are proposed for detecting line outages taking advantage of existing PMU measurements [4]–[8]. These methods are studied in our preliminary work and it is observed that LASSO based sparse estimation works successfully for a high percentage of simulated line outage cases [9].

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In this paper, the success rate of the proposed method is further improved by an orthogonal transformation. Furthermore, a final ordinary least squares estimator is used to obtain more accurate virtual current estimates of individual phases after the disconnected line is identified by LASSO estimator in three-phase networks.

The two significant contributions of the paper are:

- Improved line outage detection performance while using the same number of PMU measurements,
- Reduced computational burden without impacting accuracy.

II. Review of Equivalent Line Outage Model

Here, the equivalent line outage model will be briefly reviewed. The well-known DC power flow model is commonly used in the literature since it provides a reasonably accurate linear approximation between real power injections and voltage phase angle [6]–[8], [10]. It yields the linear formulation relating the changes in bus phase angles $\Delta\theta$ to the changes in real power injections ΔP as described in [11]:

$$B\Delta\theta = \Delta P \tag{1}$$

where,

- $B = \begin{cases} \text{if} \quad k \neq j \rightarrow B_{kj} = -\frac{1}{x_{kj}} \\ \text{if} \quad k = j \rightarrow B_{kk} = \sum_{j=1,j\neq k}^{N} \frac{1}{x_{kj}} \\ 0 \quad otherwise \end{cases}$
- x_{kj} is the branch reactance value between bus k and bus j,
- $\Delta\theta$ is the vector of phase angle differences between post outage theta and pre outage conditions,
- ΔP is the vector of bus injection changes between post and pre outage conditions.

Considering a single line outage, B and $\Delta\theta$ will alter to represent the change in the network model, assuming that loads and generation remain constant throughout the outage event. These changes can be mimicked by introducing the virtual power injections at the terminal buses of disconnected

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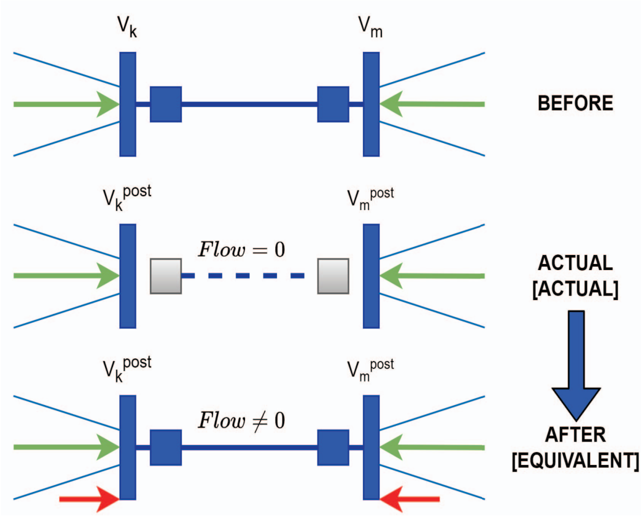

Fig. 1: Pre and Post contingency flow on branch k-m.

line without physically removing the line from the system, i.e without modifying the B matrix [12]. This line outage model is illustrated in Fig. 1.

In Fig. 1, red arrows indicates the virtual power injections at the terminal buses of disconnected line, which have same magnitude but opposite sign. In addition to that, the magnitude of these injections will be equal to the real power flow through the branch so that the effective flow will be canceled and yield net zero power flow through the branch. As a result, post-contingency state for line outage event is mimicked by only altering the terminal power injections as:

$$\Delta P_k = -\Delta P_m = \tilde{p}_{km} \tag{2}$$

where,

- ΔP_k is the virtual real power injection at bus k,
- ΔP_m is the virtual real power injection at bus m,
- $\tilde{p}_{km}$ is the virtual real power flow along the disconnected branch.

III. Line Outage Detection in Three Phase Network

In our previous work, the equivalent model of the line outage is used to detect line outages in transmission systems [9]. However, to handle phase outages in three-phase systems, the network model is transformed from phase domain to modal (sequence) domain as below.

First, PMU measurements are obtained for the pre and post outage operating conditions. Then, both the measurements as well as the bus admittance matrix are transformed into their modal (sequence) domain.

Next, the disconnected line is identified by using the method detailed in our preliminary work using only the positive sequence network.

In the final stage, the "zero, positive, and negative" sequence virtual current injections are estimated using the LASSO method based on the location of the outaged line found in the second step. Then, virtual currents of each phase for disconnected line are calculated by simply transforming these virtual sequence currents back to the phase domain. Thus, in addition to the location of the outage, the phases corresponding to the outage are also identified.

A. Transforming Measurements into Sequence Domain

At all PMU buses, voltage phasor measurements are gathered for the pre-outage and-post outage conditions. Then, the currently available bus admittance matrix and the collected measurements are transformed into symmetrical components by the transformation [13] below:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = T^{-1} * \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = T^{-1} * \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4}$$

$$Y^{012} = T^{-1} * Yabc * T \tag{5}$$

where,

- $0, 1, 2$ represent the "zero, positive, and negative" sequence networks respectively,
- a, b, c represent the "Phase A, Phase B, and Phase C" respectively,
- T is the transformation matrix given by:

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{6}$$

where, $a = 1\angle 120$.

B. Detection and Identification of the Line Outage

Once the voltage phasors are transformed from phase domain to sequence domain for both the pre and post outage scans, the method described in [9] is used only for the positive sequence components. This step is mainly used for locating the line outage rather than identifying which phases are affected.

C. Calculating Virtual Three Phase Current Injections

In the final step of line outage detection in three phase networks, instead of using the DC power flow model, the virtual current injections are calculated using QR decomposition of the network's admittance matrix Y relating bus voltages to net current injections as follows:

$$Y_{ordered} * \Delta V_{ordered} = \Delta I_{ordered} \tag{7}$$

$$Q_{ordered} * R_{ordered} * \Delta V_{ordered} = \Delta I_{ordered} \tag{8}$$

$$\begin{bmatrix} Q_{ee} & Q_{ei} \\ Q_{ie} & Q_{ii} \end{bmatrix} * \begin{bmatrix} R_{ee} & R_{ei} \\ 0 & R_{ii} \end{bmatrix} * \begin{bmatrix} \Delta V_{ee} \\ \Delta Vii \end{bmatrix} = \begin{bmatrix} \Delta I_{ee} \\ \Delta I_{ii} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} R_{ee} & R_{ei} \\ 0 & R_{ii} \end{bmatrix} * \begin{bmatrix} \Delta V_{ee} \\ \Delta Vii \end{bmatrix} = \begin{bmatrix} Q_{ee}^T & Q_{ie}^T \\ Q_{ei}^T & Q_{ii}^T \end{bmatrix} * \begin{bmatrix} \Delta I_{ee} \\ \Delta I_{ii} \end{bmatrix} \tag{10}$$

$$R_{ii} * \Delta V_{ii} = [Q_{ei}^T Q_{ii}^T] * \begin{bmatrix} \Delta I_{ee} \\ \Delta I_{ii} \end{bmatrix} \tag{11}$$

where,

- subscript i refers to buses with PMUs and,
- subscript e refers to buses without PMUs,
- ΔV is the voltage differences between post and pre outage event,
- ΔI is the virtual current injection vector,

The LASSO method can now be used to solve the sparse unknown vector ΔI, which represents the changes in the net current injections (notation is simplified by removing the "ordered" subscript) as follows:

$$\Delta I := \min_{\Delta I} \frac{1}{2} ||\Delta V' - M * \Delta I||_2^2 + \lambda ||\Delta I||_1 \tag{12}$$

where,

- $\Delta V' = R_{ii} * \Delta V_{ii}$,
- $M = \begin{bmatrix} Q_{ei}^T & Q_{ii}^T \end{bmatrix}$.

Considering that (12) consists of a complex number and Matlab's LASSO algorithm can not process complex numbers, (12) is transformed into strictly real problem using the real and imaginary parts of the equations [14]:

$$\Delta I := \min_{\Delta I} \frac{1}{2} || \begin{bmatrix} \Delta V'_{re} \\ \Delta V'_{im} \end{bmatrix} - \begin{bmatrix} M_{re} & -M_{im} \\ M_{im} & M_{re} \end{bmatrix} * \Delta I||_2^2 + \lambda ||\Delta I||_1 \tag{13}$$

where,

- Subscript re represents the real part of the matrix,
- Subscript im represents the imaginary part of the matrix.

In order to estimate the virtual current injections for individual phases, first QR transformation shown in (8) is carried out separately for "zero, positive, and negative" components replacing Y by Y_{seq}, ΔV by ΔV_{seq}, and ΔI by ΔI_{seq} as follows:

$$\begin{bmatrix} Y_{seq,ee} & Y_{seq,ei} \\ Y_{seq,ie} & Y_{seq,ii} \end{bmatrix} * \begin{bmatrix} \Delta V_{seq,e} \\ \Delta V_{seq,i} \end{bmatrix} = \begin{bmatrix} \Delta I_{seq,e} \\ \Delta I_{seq,i} \end{bmatrix} \tag{14}$$

Once (13) is formed for "zero, positive and negative" sequences separately, LASSO is performed to find virtual current injections. To identify virtual current injections, the locations of the real part and imaginary part corresponding to the solution is checked with the actual outage location found in the second step and if any of these locations do not match with the actual outage location, the corresponding column is removed from the matrix M in (13). This process is repeated until both the real part and the imaginary part locations match with the actual outage.

Finally, using the transformation matrix T, the estimated sequence currents are converted back to phase domain. Thus, the virtual current injections in each phase of the terminal buses of the disconnected branch are calculated.

IV. The Final Improvement via Ordinary Least Squares Estimation

Although most of the time accuracy of the sparse estimation method is quite satisfactory, there may be situations where iteratively estimated virtual current injections using LASSO may not be sufficiently accurate due to the co-linearity of the coefficient matrix columns. Therefore, estimation of the virtual current injections can be formulated as a standard least squares estimation problem once the location of the disconnected line is already identified. Estimation can be carried out for each sequence component as follows:

$$[Q_{bus1,seq}^{outage} \; Q_{bus2,seq}^{outage}] * \begin{bmatrix} \Delta I_{bus1,seq}^{outage} \\ \Delta I_{bus2,seq}^{outage} \end{bmatrix} = \Delta V'_{seq} \tag{15}$$

where,

- Subscript seq refers to "zero, positive, and negative" sequences,
- $Q_{bus1,seq}^{outage}$ and $Q_{bus2,seq}^{outage}$ denotes the reduced $[Q_{ei}^T \; Q_{ii}^T]$ matrix for the corresponding sequence, whose columns correspond to the terminal buses of the disconnected branch and whose rows correspond to the buses with PMU measurements,
- $\Delta I_{bus1,seq}^{outage}$ and $\Delta I_{bus2,seq}^{outage}$, are the entries of the 2×1 unknown current vector for the corresponding sequence, at the terminal buses of the disconnected branch,
- $\Delta V'_{seq} = R_{ii,seq} * \Delta V_{ii,seq}$,

The estimation problem of (15) can be solved using a standard Ordinary Least Squares (OLS) algorithm [15]:

$$\hat{\beta} = (X^T X)^{-1} X^T y \tag{16}$$

where:

$\beta = \begin{bmatrix} \Delta I_{bus1,seq}^{outage} \\ \Delta I_{bus2,seq}^{outage} \end{bmatrix}$,

$y = \Delta V'_{seq}$,

$X = [Q_{bus1,seq}^{outage} \; Q_{bus2,seq}^{outage}]$.

By solving (16) for "zero, positive and negative" sequences, virtual current injections are calculated more accurately in a single iteration. Then, these current injections are transformed back to the phase domain using the transformation matrix T in (6) to determine the virtual current injections at each individual phase.

V. Simulation Results

To compare the performances of the proposed method and the sparse estimation method in finding virtual current injections, 127-bus three-phase unbalanced distribution system is created as in Fig. 2. The system shown in Fig 2 consists of 134 three phase branches and the ”R/X” ratios of these branches vary between ”0.4 to 1”. Once the distribution system with a high ”R/X” ratio is created, each method is tested on a 127-bus three-phase unbalanced distribution system assuming that a small number of PMU measurements are available. Moreover, each PMU is assumed to measure the voltage phasor at the bus and the current phasor along one of the incident branches to that bus. These current phasors enable calculation of the voltage phasors at the remote end of the respective branches. These PMU locations are colored in red in Fig. 2.

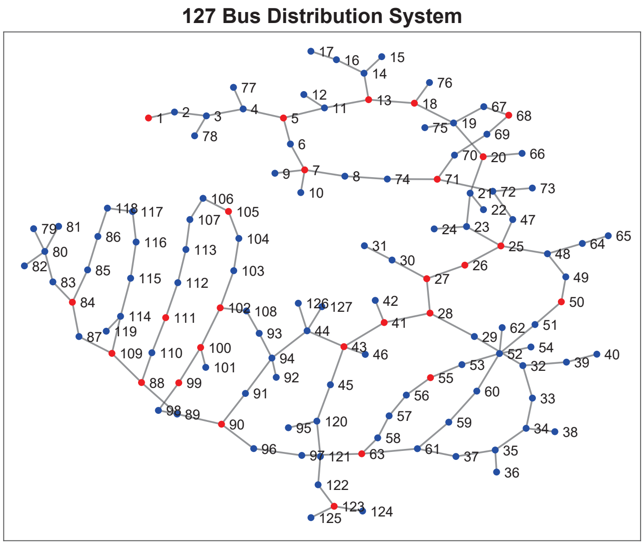

Fig. 2: 127-bus Three-phase Distribution System Diagram.

Once PMUs are located across the three phase distribution system, cases in Table I are created and tested with using the sparse estimation method and the proposed method.

Initially, the location of the line outage is determined using only the positive sequence network and the measurements as described in the second step of the strategy. The results, of detected line outage locations for the cases in Table I are given in Table II. In Table II, ”$\widetilde{P}_{st}$ and $\Delta\widetilde{P}$” represent the virtual real power flow along the disconnected branch calculated using the Power Transfer Distribution Factors (PTDF) and the virtual power flow calculated using the proposed method, respectively.

Once the location of the outage is determined, in the second stage, ”zero, positive, and negative” sequence currents are calculated using the sparse estimation method and the proposed method based on the found location in Table II. Then, to determine the detailed phase outages, these sequence currents are transformed back into the phase domain. The results of the virtual current injections of each phase at the terminal buses of disconnected line are given in Table III.

Considering the results in Table III, solely using LASSO estimation method is shown to fail to identify the true phase outages for the highlighted cases. Use of the proposed OLS estimation helps to accurately identify the virtual current injections of each phase for the disconnected branch. In addition, since sparse estimation iterations are avoided, the proposed approach will be computationally more efficient.

TABLE I: Simulation cases for the three phase distribution system.

Case Number	Disconnected Line (From Bus-To Bus)	Disconnected Phase
1	87-109	A, B, C
2	87-109	A, C
3	87-109	B, C
4	87-109	A, B
5	19-67	A, B, C
6	19-67	A, C
7	19-67	B, C
8	19-67	A, B
9	71-72	A, B, C
10	71-72	A, C
11	71-72	B, C
12	71-72	A, B
13	44-94	A, B, C
14	44-94	A, C
15	44-94	B, C
16	44-94	A, B
17	25-47	A, B, C
18	25-47	A, B
19	25-47	A, C
20	25-47	B, C

TABLE II: The results of the found locations for the line outages in Table I.

Outage Line Branch / Phase / $\widetilde{P}_{st}$	Detected Outage Iteration No. / Branch / $\Delta\widetilde{P}$
87-109 / A, B, C / 2.24	1 / 87-10 / 2.23
87-109 / A, B / 2.24	1 / 87-10 / 1.84
87-109 / A, C / 2.24	1 / 87-10 / 1.61
87-109 / B, C / 2.24	1 / 87-10 / 1.79
19-67 / A, B, C / 9.48	1 / 19-67 / 7.92
19-67 / A, B / 9.48	1 / 19-67 / 6.57
19-67 / A, C / 9.48	1 / 19-67 / 6.51
19-67 / B, C / 9.48	1/ 19-67 / 6.71
71-72 / A, B, C / 33.08	1 / 71-72 / 42.85
71-72 / A, B / 33.08	1 / 71-72 / 32.11
71-72 / A, C / 33.08	1 / 71-72 / 32.16
71-72 / B, C / 33.08	1 / 71-72 / 32.30
44-94 / A, B, C / 1.94	1 / 44-94 / 2.42
44-94 / A, B / 1.94	1 / 44-94 / 1.74
44-94 / A, C / 1.94	1 / 44-94 / 1.74
44-94 / B, C / 1.94	1 / 44-94 / 1.99
25-47 / A, B, C / -52.97	1 / 25-47 / -45.33
25-47 / A, B / -52.97	1 / 25-47 / -34.23
25-47 / A, C / -52.97	1 / 25-47 / -34.28
25-47 / B, C / -52.97	1 / 25-47 / -34.54

TABLE III: Magnitude of the virtual current injections of each phase for the disconnected branch.

Outage Line	Sparse Estimation Method				Proposed Method			
	Phase A (p.u)	**Phase B (p.u)**	**Phase C (p.u)**	**Time (ms)**	**Phase A (p.u)**	**Phase B (p.u)**	**Phase C (p.u)**	**Time (ms)**
87-109 / A, B, C	1.7972	2.0295	1.6095	∼118	1.7962	2.0317	1.6081	∼8
87-109 / A, B	1.9598	1.8864	0	∼118	1.9622	1.8911	0	∼8
87-109 / A, C	1.6004	0	1.8451	∼118	1.6030	0	1.8495	∼8
87-109 / B, C	0	2.1849	1.5338	∼118	0	2.1908	1.5396	∼8
19-67 / A, B, C	4.0548	4.4252	4.0360	∼125	4.0538	4.4245	4.0351	∼8
19-67 / A, B	3.9662	4.8718	0	∼125	3.9653	4.8712	0	∼8
19-67 / A, C	4.6479	0	3.8161	∼125	4.6470	0	3.8154	∼8
19-67 / B, C	2.8142	4.1046	3.4257	∼125	0	4.3013	4.5354	∼8
71-72 / A, B, C	40.1950	40.1171	40.2364	∼221	41.0264	41.6133	41.1074	∼8
71-72 / A, B	33.8520	34.8423	23.5645	∼221	40.1279	38.6688	0	∼8
71-72 / A, C	33.5436	22.7711	36.1345	∼221	38.0969	0	40.1691	∼8
71-72 / B, C	0	39.3532	37.1193	∼221	0	40.7144	38.2415	∼8
44-94 / A, B, C	1.9897	2.7437	2.2898	∼59	1.9697	2.7558	2.2866	∼8
44-94 / A, B	2.1501	2.4516	0	∼59	2.1368	2.4721	0	∼8
44-94 / A, C	1.6969	0	2.4704	∼59	1.70550	0	2.4857	∼8
44-94 / B, C	0	2.8296	2.1028	∼59	0	2.8423	2.1057	∼8
25-47 / A, B, C	29.2387	29.7093	29.3713	∼100	29.2387	29.7141	29.3677	∼8
25-47 / A, B	27.0325	29.577	0	∼100	27.0343	29.5770	0	∼8
25-47 / A, C	29.1038	0	27.0884	∼100	29.0927	0	27.1688	∼8
25-47 / B, C	0	27.5821	29.3302	∼100	0	27.6372	29.3412	∼8

VI. Conclusion

Accurate detection of disconnected phases in three phase networks is as important as finding the location of the outaged line. This paper presents an improved method for accurately determining disconnected phases or entire lines in a three-phase unbalanced network. The main advantage of the proposed method is that it relies on a limited number of PMU measurements to detect line outages in the entire system. The method is explained in detail along with simulation results for a typical three-phase distribution system. Future work will involve extension of the proposed method to systems with branches having non-symmetric conductor configurations.

References

[1] U.-C. P. S. O. T. Force, "Final report on the august 14, 2003 blackout in the united states and canada," *Dept. Energy*, Apr. 2004 [Online] https://www.energy.gov/sites/default/files/oeprod/DocumentsandMedia/BlackoutFinal-Web.pdf.

[2] FERC and NERCH, "Arizona-southern california outages on september 8, 2011: Causes and recommendations," Apr. 2012.

[3] N. A. E. R. Corporation., "Scada failure resulting in loss of monitoring function," [Online] http://www.nerc.com/files/LL20130203SCADAFailureResultinginLossofMonitoringFunction.pdf.

[4] S. Azizi, M. R. Jegarluei, A. M. Kettner, and A. S. Dobakhshari, "Wide-area line outage monitoring by sparse phasor measurements," *IEEE Transactions on Power Systems*, pp. 1–11, 2022 (Early Access).

[5] Y. Liao, Y. Weng, C.-W. Tan, and R. Rajagopal, "Quick line outage identification in urban distribution grids via smart meters," *CSEE Journal of Power and Energy Systems*, vol. 8, no. 4, pp. 1074–1086, 2022.

[6] J. E. Tate and T. J. Overbye, "Line outage detection using phasor angle measurements," *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1644–1652, 2008.

[7] H. Zhu and G. B. Giannakis, "Sparse overcomplete representations for efficient identification of power line outages," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2215–2224, 2012.

[8] B. Donmez and A. Abur, "Sparse estimation based external system line outage detection," in *2016 Power Systems Computation Conference (PSCC)*, pp. 1–6, 2016.

[9] T. Yildiz and A. Abur, "Improved line outage detection in transmission systems with few PMUs," in *2022 North American Power Symposium (NAPS)*, pp. 1–5, 2022.

[10] R. Emami and A. Abur, "External system line outage identification using phasor measurement units," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1035–1040, 2013.

[11] K. Purchala, L. Meeus, D. Van Dommelen, and R. Belmans, "Usefulness of DC power flow for active power flow analysis," in *IEEE Power Engineering Society General Meeting, 2005*, pp. 454–459 Vol. 1, 2005.

[12] A. J. Wood, B. F. Wollenberg, and G. B. Sheblé, *Power generation, operation, and control*. John Wiley & Sons, 2013.

[13] A. R. Bergen and V. Vittal, *Power System Analysis*. Prentice-Hall, 2000.

[14] "Regularized least-squares regression using lasso or elastic net algorithms," MathWorks. [Online]. Available: http://www.mathworks.com/help/stats/lasso.html.

[15] C. Dismuke and R. Lindrooth, "Ordinary least squares," *Methods and Designs for Outcomes Research*, vol. 93, pp. 93–104, 2006.