2023 IEEE Power & Energy Society General Meeting (PESGM) | 978-1-6654-6441-3/23/$31.00 ©2023 IEEE | DOI: 10.1109/PESGM52003.2023.10252202

Estimation of Participation Factors Using the Synchrosqueezed Wavelet Transform

Mahsa Sajjadi, Tianwei Xia, Min Xiong, Kai Sun
Department of EECS
University of Tennessee, Knoxville, TN, USA
msajjad1@vols.utk.edu, txia4@vols.utk.edu,
mxiong3@vols.utk.edu, kaisun@utk.edu

Andy Hoke, Jin Tan
Power Systems Engineering Center
National Renewable Energy Laboratory
andy.hoke@nrel.gov, Jin.Tan@nrel.gov

Bin Wang
Department of ECE
University of Texas at San Antonio, TX, USA
bin.wang2@utsa.edu

***Abstract*—This paper proposes a data-driven approach for estimating participation factors for a power system using only simulation results on selected disturbances. The approach is purely response-based and does not need a linearized system model for eigen-analysis, which makes it applicable to systems whose detailed, complete mathematical models are not available. Considering the unavoidable nonlinearity as exhibited in the transient period of a system response, the Synchrosqueezed Wavelet Transform is applied to simulated responses for modal analysis to obtain participation factors. Based on simulations of Kundur's two-area system using both the electromagnetic transient model and phasor model, the participation factors estimated by the proposed approach are compared with two other signal processing tools, the Prony analysis and continuous wavelet transform, and are also benchmarked with conventional model-based participation factors.**

***Index Terms*—Participation factor, Power system oscillation, Prony analysis, Synchrosqueezed wavelet transform, Wavelet transform**

I. Introduction

The stability and dynamic performance of a power system are threatened by sustained power oscillations, particularly those that occur after a large disturbance. To effectively control the poorly damped or even growing oscillations of power systems, the generators that significantly participate in the oscillation should be identified. Participation Factors (PFs) were first introduced as a technique to quantify the relative participation of oscillation modes in each state variable of a generator and of state variables in oscillation modes [1-3]. Participation factors have been used extensively in electrical power systems and other areas as a powerful index for stability analysis [4], coherency and clustering, model reduction [5], controller and sensor placement [6, 7], etc.

Traditional model-based methods for modal properties and participation factors are based on eigen-analysis of a linearized system model. However, the detailed, complete mathematical model of power system may not be always available especially for the Inverter-Based Resources (IBRs) and their associated controllers. For instance, manufacturers would only offer the black-box models of IBRs during the power system planning studies. These devices and control functions have only black-box models due to, e.g., confidentiality. Time-domain simulation of such components together with the rest of the system can be conducted without any issue to provide responses of the system subject to any disturbance. However, their linear, mathematical models can hardly be obtained for eigen-analysis [8, 9]. [9] uses the black-box model of IBR to conduct small signal stability analysis. Several small-signal positive and negative voltage perturbations with different frequencies are applied to the point of common coupling to obtain the impedance of the IBR model [9]. The data-driven approaches to calculate participation factors are presented in [10, 11]. [11] develops a data-driven approach based on the Koopman mode decomposition. This approach needs some certain conditions are satisfied, e.g. it requires the components of the initial state vector follow a uniform probability density function and they should be statistically independent with zero mean.

This paper proposes a data-driven approach for estimating participation factors for a power system only from simulation results on selected disturbances. The approach is purely response-based and hence does not need a linearized system model of the whole power system for eigen-analysis. The approach can be applied to systems whose detailed, complete mathematical models are unavailable. Considering possible nonlinear dynamics of a power system as exhibited in the transient period of its response under a disturbance, the Synchrosqueezed Wavelet Transform (SSWT) is applied to simulated responses for modal analysis to obtain estimated participation factors. The SSWT method was originally developed to analyze audio signals, whose modal properties can vary continuously with time [12].

In this paper, simulations are conducted on both the Electromagnetic Transient (EMT) model and the phasor model of Kundur's two-area system. Estimated PFs from these two models are compared. For electromechanical oscillations, comparison results show a close resemblance of the modal properties from EMT simulations to those from phasor simulations. The PF results using SSWT are also compared with Prony analysis and Continuous Wavelet Transform (CWT) methods [12, 13]. SSWT can make the spectrum of CWT sharper and clearer, improve the frequency resolution, and result in more accurate detection of mode frequency than the Prony analysis and CWT methods. The paper also benchmarks the estimated PFs

This material is based upon work supported by the U.S. Department of Energy's Office of Energy Efficiency and Renewable Energy (EERE) under the Solar Energy Technologies Office Award Number 38457. The views expressed herein do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

from the proposed approach to the PFs calculated by eigen-analysis on the linearized phasor model. The accuracy of results is confirmed.

In the rest of this paper, Section II briefly describes participation factor concept. In Section III, the proposed response-based approach for modal analysis is presented and related signal processing techniques are introduced. In Section IV, the participation factor results for Kundur's two-area system using the preferred SSWT are compared with the results from the Prony analysis and continuous wavelet transform methods, and are also benchmarked with the PFs calculated from the model. Finally, the conclusion is drawn in Section V.

II. Preliminary of Participaton Factors

A set of nonlinear, first-order differential equations can be used to represent a power system dynamic model as the following form [1].

$$\dot{x} = f(x,u) \tag{1a}$$

$$y = g(x,u) \tag{1b}$$

The vector x is referred as the state vector, whose entries are state variables of the system. The vectors u and y are the input and output vector of the system, respectively. g shows nonlinear functions which connect state variables and inputs to the system outputs.

The equilibrium point of the system can be obtained by letting the derivatives of all state variables be zeros simultaneously. At the equilibrium point all state variables are constant and unaffected by time [1].

For a sufficiently small deviation of the system position from its equilibrium point, the nonlinear system can be approximated by the first term of Taylor's series expansion as the following forms:

$$\Delta\dot{x} = A\Delta x + B\Delta u \tag{2a}$$

$$\Delta y = C\Delta x + D\Delta u \tag{2b}$$

where A, B, C, and D are matrices that represent partial derivatives of the functions f and g with respect to the state vector x and the input vector u [1].

The eigenvalues of A matrix, denoted as λ_i, i=1, 2,…, can be determined by the characteristic equation in the following form :

$$\det(\mathbf{A} - \lambda_i \mathbf{I}) = 0 \tag{3}$$

For any eigenvalue λ_i, the vectors $\mathbf{\Phi}_i$ and $\mathbf{\Psi}_i$ which satisfy equations (4a) and (4b) are called right and left eigenvectors of **A** associated with the eigenvalue λ_i [1].

$$\mathbf{A\Phi}_i = \lambda_i \mathbf{\Phi}_i \tag{4a}$$

$$\mathbf{\Psi}_i \mathbf{A} = \lambda_i \mathbf{\Psi}_i \tag{4b}$$

where $\mathbf{\Phi}_i$ is n-column vector and $\mathbf{\Psi}_i$ is n-row vector, n is the number of eigenvalues [1].

Right and left eigenvectors depend on the units of state variables. As a solution, a dimensionless criterion named linear participation factor is defined as a combination of right and left eigenvectors as:

$$p_{ki} = \mathbf{\Phi}_{ki} \mathbf{\Psi}_{ik} \tag{5}$$

where p_{ki} is the participation of the kth state variable in the ith mode, and vice versa [1].

III. Response-based Participation Factors Using Wavelets

This section first introduces a response-based approach for PFs estimation and then introduces SSWT. The proposed response-based approach uses the SSWT to obtain modal properties from simulated responses starting from selected initial conditions.

Consider the zero-input response of system in (2a). To eliminate the cross-coupling between state variables, the linear change of coordinate as (6a) is considered which transforms the system to its Jordan form as (6b) shows [1].

$$x = \mathbf{\Phi}_{ki} z \tag{6a}$$

$$\dot{z} = \Lambda z \tag{6b}$$

where Λ is a diagonal matrix and $\Lambda = \Phi^{-1} A \Phi$. The response of system in terms of the original state becomes:

$$x_k(t) = \sum_{i=1}^{n} \Phi_{ki}\, \Psi_i\, x(0) e^{\lambda_i t} = \sum_{i=1}^{n} B_{ki} e^{\lambda_i t} \tag{7}$$

where x(0) is the initial condition and B_{ki} is the contribution factor.

The physical meaning of participation factor for kth state variable is the magnitude of modal oscillation in a machine state when only that state variable is perturbed [14]. In this sense, the vector of initial condition is $x_0 = e_k$, as (8) shows.

$$\mathbf{x}_0 = \mathbf{e}_k \overset{\text{def}}{=} \begin{bmatrix} 0 & \dots & 0 & \underset{k\text{th element}}{1} & 0 & \dots & 0 \end{bmatrix}^{\mathrm{T}} \tag{8}$$

By inserting (8) into (7) the system response for kth state variable becomes:

$$x_k(t) = \sum_{i=1}^{n} \Phi_{ki}\, \Psi_i\, e_k e^{\lambda_i t} = \sum_{i=1}^{n} p_{ki} e^{\lambda_i t} \tag{9}$$

Therefore, to calculate the response-based participation factor, state variables should be perturbed one by one and by small amount so that the magnitude of oscillation obtained from wavelets shows linear participation factor for the corresponding perturbed state variable.

The continuous wavelet transform for a given signal $s(t)$, is defined as follows:

$$W(a,b) = \int_{-\infty}^{+\infty} s(t) a^{-1/2} \Psi^*\left(\frac{t-b}{a}\right) dt \tag{10}$$

where a and b are time scale factor and time shift factor, respectively. Ψ^* represents complex conjugate of mother wavelet $\Psi(t)$. The instantaneous frequency for each point in the plane can be obtained using:

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$$\omega(a,b) = -iW(a,b)^{-1}\frac{\partial W(a,b)}{\partial b} \tag{11}$$

where ω (*a*, *b*) shows instantaneous frequency of each point of plane [13].

Synchrosqueezed wavelet transform method is a signal processing tool based on the time-frequency decomposition. This method was originally offered to tackle a low-resolution spectrum issue generated by the traditional Time-Frequency (TF) analysis tools. The energy spectrum of continuous wavelet is always spread out in the TF plane, which results in a blurry time frequency representation [13]. However, SSWT squeezes wavelet coefficients of the same instantaneous frequency close to the central frequency, which results in a sharper and more concentrated TF energy spectrum [13].

By the recombination of continuous wavelet coefficients, the synchrosqueezed wavelet transform can be defined as:

$$SSWT(\omega_l, b) = (\Delta\omega)^{-1} \sum_{a_k:|\omega(a_k,b)-\omega_l|\le \Delta\omega/2} W(a_k,b)a_k^{-3/2}(\Delta a)_k \tag{12}$$

where a_k is scale. Both Δa and $\Delta\omega$ are discrete with a_k-a_{k-1}=Δa_k and ω_l-ω_{l-1}=$\Delta\omega$. Synchrosqueezed transform SSWT(ω_l, *b*) is determined only at the centers ω_l of the successive bins [ω_l-0.5$\Delta\omega$, ω_l+0.5$\Delta\omega$] [13].

The outputs of wavelet transforms include: 1) frequencies of the wavelet transform, returned as a vector with dimension of N_a and 2) an $N_a \times N$ matrix, as the magnitude of oscillations. N_a is the number of frequencies and *N* is the number of sampling time. Therefore, the outputs of wavelet transforms are three-dimensional time-frequency representations including the magnitude of oscillations in each corresponding frequency and sample time. The magnitude of oscillations in each moment shows the participation factor value.

IV. Simulation Results

Kundur's two-area system with 4 machines and 11 buses is selected as the test case [1]. Fig.1 shows the topology of this system.

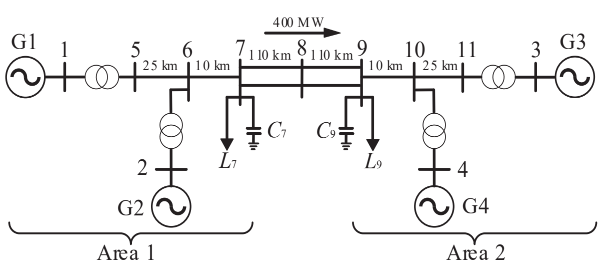

Figure 1. Kundur's two-area system

A. Simulation Results from EMT and Phasor Models

As mentioned in Section III, to obtain the response-based participation factors, the first step is to perturb a state variable of a generator. The trajectory of the selected state variable over time is then obtained to apply modal analysis. This process is repeated for all generators. Figs. 2 and 3 show the rotor speed deviation's trajectories of four generators after perturbation for the EMT and phasor responses over a time window of t=0-10 seconds. The time step for the phasor responses is 5 ms and for the EMT responses is 70 µs. The comparison of rotor speed deviations of EMT and phasor responses in Figs. 2 and 3 shows very similar trajectories for each generator and therefore these rotor speed deviation's trajectories can be expected to have a similar modal property.

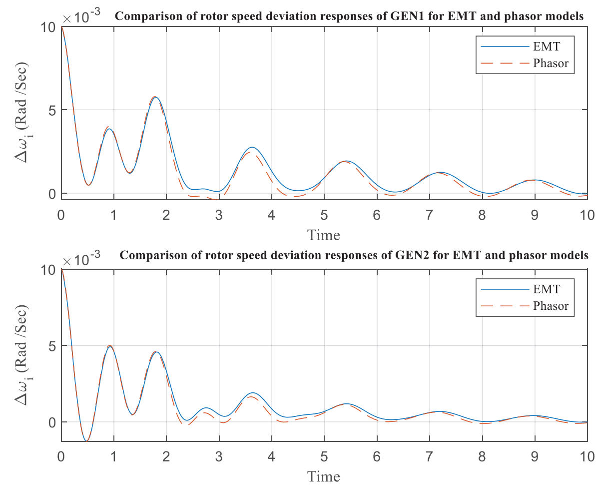

Figure 2. Rotor speed deviation's trajectories of Gen1 and Gen2 for EMT and phasor models

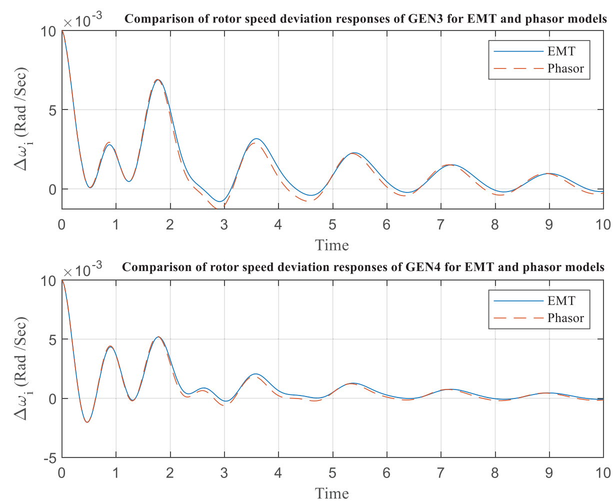

Figure 3. Rotor speed deviation's trajectories of Gen3 and Gen4 for EMT and phasor models

The SSWT and CWT are applied to each rotor speed deviation's trajectory in Figs. 2 and 3 to estimate modal information of the system. Fig. 4 shows the SSWT and CWT spectrums of the EMT response for Gen3. The SSWT and CWT spectrums of the phasor response for Gen3 are also presented in Fig. 5. The SSWT and CWT of a signal are three-dimensional spectra in which the x-axis, y-axis and z-axis show respectively time, frequency, and amplitude of the signal in each moment. Two modes are observable in Figs. 4 and 5, one with a frequency in the range of 1.0-1.5 Hz and the second with a frequency around 0.5 Hz. Comparing Figs. 4 and 5, the SSWT for EMT and phasor

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responses show almost the same time-frequency spectrum. The same is true for the CWT spectrums of the EMT and phasor responses.

From Figs. 4 and 5, the time-frequency spectrum of SSWT is sharper and more focused compared to that of the CWT, which results in easier estimation of mode-frequency and PFs. The magnitude of SSWT spectrum after a couple of seconds of simulation is considered to estimate PFs because the transient and edge effects have almost disappeared after few seconds. For the CWT, the center value of the spectrum is considered to obtain mode-frequency and participation factors. The center frequency's value is the midpoint between the frequencies on either side of the peak. The normalized participation factor results of EMT and phasor responses are included in Table I and II for the SSWT and CWT methods, respectively. The normalized participation factors <0.001 are approximated by zero in the Tables I and II.

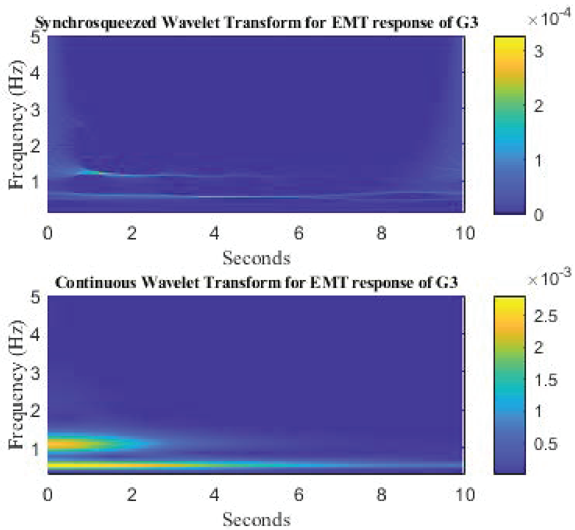

Figure 4. SSWT and CWT spectrum of EMT response for G3

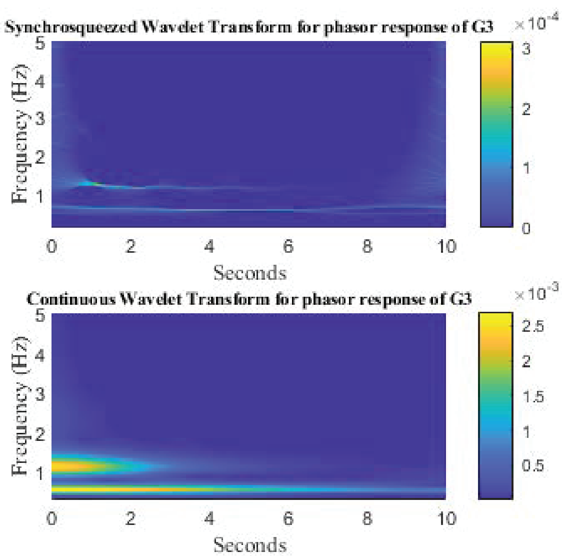

Figure 5. SSWT and CWT spectrum of phasor response for G3

The response-based modal results in Tables I, II, and III show that the two-area system has three oscillation modes. Each area of the system has one local mode, for area 1 around 1.1 Hz, for area 2, around 1.2 Hz, and an inter-area mode with a frequency around 0.6 Hz. For the inter-area mode, the generators 1 and 2 are oscillating against generators 3 and 4 and all generators show large participation in the mode, however for the two local modes only the generators in each area show larger participations according to the results. As Table I shows, the EMT-based PF results obtained by SSWT have close agreement with the phasor-based PF results, which means that phasor model can appropriately capture the dynamics of electromechanical modes in the EMT model. EMT-based and phasor-based PF results obtained by the CWT and Prony approaches are also close.

TABLE I. PF RESULTS FOR EMT AND PHASOR USING SSWT

Gen.	Inter-area mode 0.565 Hz		Local mode 1 ~ 1.10 Hz		Local mode 2 ~1.2 Hz	
	EMT	Phasor	EMT	Phasor	EMT	Phasor
1	0.685	0.671	0.739	0.733	~ 0	~ 0
2	0.328	0.326	**1.000**	**1.000**	~ 0	~ 0
3	**1.000**	**1.000**	~ 0	~ 0	0.511	0.504
4	0.471	0.454	~ 0	~ 0	**1.000**	**1.000**

TABLE II. PF RESULTS FOR EMT AND PHASOR USING CWT

Gen.	Inter-area mode 0.550 Hz		Local mode 1 ~1.10 Hz		Local mode 2 ~1.2 Hz	
	EMT	Phasor	EMT	Phasor	EMT	Phasor
1	0.724	0.726	0.747	0.743	~ 0	~ 0
2	0.463	0.416	**1.000**	**1.000**	~ 0	~ 0
3	**1.000**	**1.000**	~ 0	~ 0	0.674	0.691
4	0.578	0.542	~ 0	~ 0	**1.000**	**1.000**

TABLE III. PF RESULTS FOR EMT AND PHASOR USING PRONY

Gen.	Inter-area mode 0.560 Hz		Local mode 1 ~1.10 Hz		Local mode 2 ~ 1.2 Hz	
	EMT	Phasor	EMT	Phasor	EMT	Phasor
1	0.714	0.707	0.507	0.686	0.158	0.066
2	0.381	0.390	**1.000**	**1.000**	0.111	0.133
3	**1.000**	**1.000**	0.402	0.388	0.540	0.566
4	0.500	0.512	0.059	0.100	**1.000**	**1.000**

B. *Model-Based PFs Results*

The modal information results obtained from small-signal linearized phasor model of the two-area system is provided in this section for the comparison purposes. A detailed six-order generator model is used to model each generator, whose six differential equations include two swing equations and four voltage equations in two axes.

Table IV shows the model-based PFs results of the phasor model for rotor speed deviation of generators. There are three oscillatory modes in the Kundur's two area system. There is an inter-area mode with the frequency of 0.564 Hz and two local modes 1.097 Hz and 1.265 Hz related to each area of the system.

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TABLE IV. MODEL-BASED PFS FROM PHASOR MODEL

Gen.	0.564 Hz	1.097 Hz	1.265Hz
1	0.5779	0.7465	0.00058
2	0.3399	**1.0000**	0.0047
3	**1.0000**	0.005097	0.5550
4	0.4769	0.000284	**1.0000**

Comparing Table IV and Tables I, II, and III for the phasor model shows that the estimated PFs results obtained by SSWT is closer to the benchmark model-based approach results. In order to compare the accuracy of estimated PFs obtained by SSWT, CWT and Prony for the phasor responses, their absolute errors from the model-based PFs are shown in Table V. To differentiate two different participation factors and not to miss the ranking, this study suggests 0.1 as a threshold. According to the Table V only the SSWT can satisfy this requirement because continuous wavelet and Prony have an absolute error more than 0.1 in some cases, which is not desired.

TABLE V. ABSOLUTE ERROR OF ESTIMATED PFS

Gen.	Inter-area mode			Local mode 1			Local mode 2		
	SSWT	CWT	Prony	SSWT	CWT	Prony	SSWT	CWT	Prony
1	0.093	0.148	0.130	0.0135	0.0035	0.0605	0.000	0.000	0.065
2	0.0139	0.076	0.0501	0.000	0.000	0.000	0.0047	0.0047	0.128
3	0.000	0.000	0.000	0.005	0.005	0.383	0.051	0.136	0.011
4	0.023	0.0651	0.0351	0.000	0.000	0.099	0.000	0.000	0.000

V. CONCLUSION

Three different response-based approaches are employed in this paper to estimate participation factors of EMT-based and phasor-based rotor speed deviation's responses for Kundur's two area system. The estimated PFs of these two models for the electromechanical modes are similar, which means that the phasor model can appropriately capture the dynamics of electromechanical modes in the EMT model. Therefore, phasor responses can be used to determine EMT model's modal properties for the electromechanical modes. This significantly reduces the computational burden because EMT simulation of a large-scale power system for detailed EMT models is exceedingly time-consuming, if not impossible.

The results of the estimated PFs utilizing SSWT are contrasted with those from Prony analysis and CWT. SSWT can enhance the CWT spectra, and result in more precise mode frequency detection. It is verified that the results are accurate by comparing the response-based results to those obtained using the conventional model-based approach for the phasor model.

For a large-scale power grid having IBRs located in multiple areas, the proposed data-driven participation factors can be used to help decide which areas highly participate in an oscillation mode of interest so that EMT simulations can be performed in those areas for more detailed dynamics of IBRs interacting with neighboring generation and transmission facilities. As reported by this paper, the accurate match between participation factors respectively from simulations on the phasor model and EMT model of a power system indicates that participation factors on electromechanical modes can be estimated from much faster simulations on the phasor model, which can accelerate the identification of highly participating areas.

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