Data-Driven Analysis of Power System Dynamic Performances via Koopman Mode Decomposition

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Challenging Issue

• How do we utilize massive quantities of data for analysis and control of multi-scale power systems?
  – Measured: voltage, phase, power flow
  – Predicted: wind speed (renewable output), demand power, EV-sharing (movement of multiple EVs with batteries in space- and time-domain)

from https://www.naspi.org/
from Nagoya Univ. HyARC
at Anjo-City, Japan
Purpose and Contents

Data-Driven Analysis of Power System Dynamic Performances

• Application of Koopman Mode Decomposition
  – A (nonlinear) generalization of linear oscillatory modes, guided by operator theory of nonlinear dynamical systems—Koopman Operator

1. Brief Summary of the Underlying Theory
2. Two Applications:
   1. Modal Identification
   2. Power Flow Diagnostic
3. Message and Ongoing Work
Koopman Mode Decomposition

Mathematical Formulation:

\[ x_{k+1} = T(x_k), \quad x \in M \]
\[ g : M \rightarrow \mathbb{R} \]
\[ U g(x) = g \circ T(x) \]

- Finite-dimensional nonlinear model, which describes internal state dynamics of a power system
- Observable or output of the model, which describes measurement or sampling of the dynamics
- Koopman operator (linear!) that describes time evolution of the measured quantity

Decomposition of Time Evolution of Vector-valued Observable:

\[ g(x_k) = \sum_i \lambda_i^k \phi_i(x_0) V_i \]

Measured quantity (multi-dim.)

Eigen-values and eigen-functions of \( U \):

\[ U \phi_i = \lambda_i \phi_i \]

Computation of Koopman Modes

**Arnoldi-like Algorithm** to compute an approximation of the Koopman eigenvalues and modes *directly from data*:

\[
\{g_0, \ldots, g_m\}
\]

Finite-time data obtained in experiments or simulations under uniform sampling

\[
g_k = \sum_{j=1}^{m} \tilde{\lambda}_j^k \tilde{V}_j, \quad g_m = \sum_{j=1}^{m} \tilde{\lambda}_j^m \tilde{V}_j + \eta_m
\]

\[k = 0, \ldots, m - 1\]

Modal Identification (1/2) - NORDEL Grid

- No similar mode is identified by linearization of the swing equation.
- The inter-area mode is inherently nonlinear!

Simulation data with nonlinear swing equation of Nordic 32-bus test system w/ 22 generators


Koopman Mode Decomposition

0.3-Hz Inter-area Mode

Susuki, Data-Driven Analysis of Power Systems via Nonlinear Koopman Modes

**Inter-area mode** (0.39 Hz “Chubu-Hokuriku-Kaisai v.s. Chugoku-Kyushu”) with small damping is detected.
Power Flow Diagnostic (1/2)

Measured Data on Power Flows in 2011 Arizona-Southern California Grid Outages


\[ P_{j,j+1}^{(k)} = \begin{bmatrix} \tilde{\lambda}_j^k \tilde{V}_j + (\tilde{\lambda}_{j+1})^k \tilde{V}_{j+1} \\ \vert \tilde{V}_{j1} \vert \cos\{2\pi k \tilde{V}_j + \text{Arg}(\tilde{V}_{j1})\} \\ \vert \tilde{V}_{j2} \vert \cos\{2\pi k \tilde{V}_j + \text{Arg}(\tilde{V}_{j2})\} \\ \vdots \\ \vert \tilde{V}_{jm} \vert \cos\{2\pi k \tilde{V}_j + \text{Arg}(\tilde{V}_{jm})\} \end{bmatrix} \]

\[ = 2\pi_j^k \begin{bmatrix} \tilde{V}_j \end{bmatrix} \]

**Koopman Modes**
A Large-scale, slow-growing mode leading to the collapse of the grid is detected.

Conclusion - Message and Ongoing Work

Message:

- Nonlinear Koopman modes enable the development of fully data-driven methodology and tools for power system analysis, which have a solid mathematical foundation---Koopman operator.

Ongoing Work:

- Data-driven decision-making w/ measured and predicted data such as
  - Wind flow field;
  - EV-sharing (in Prof. Suzuki’s Super-Team).