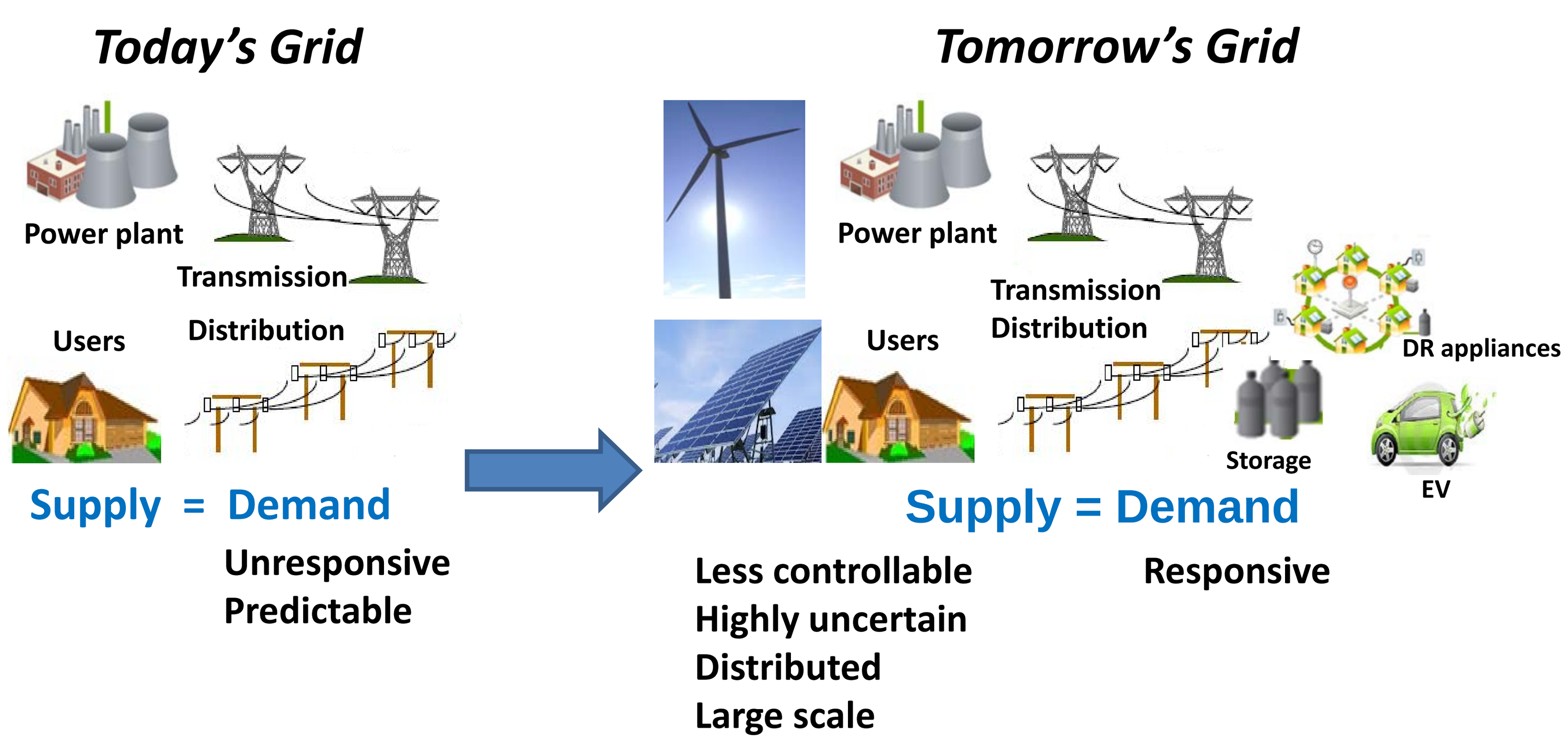
Bridging the Gap: Distributed Frequency Control and Economic Efficiency

Na (Lina) Li, *School of Engineering and Applied Sciences (SEAS), Harvard University*

Motivation



Challenge: **The control and optimization for power networks need to operate at faster time-scales for reliability and economic efficiency.**

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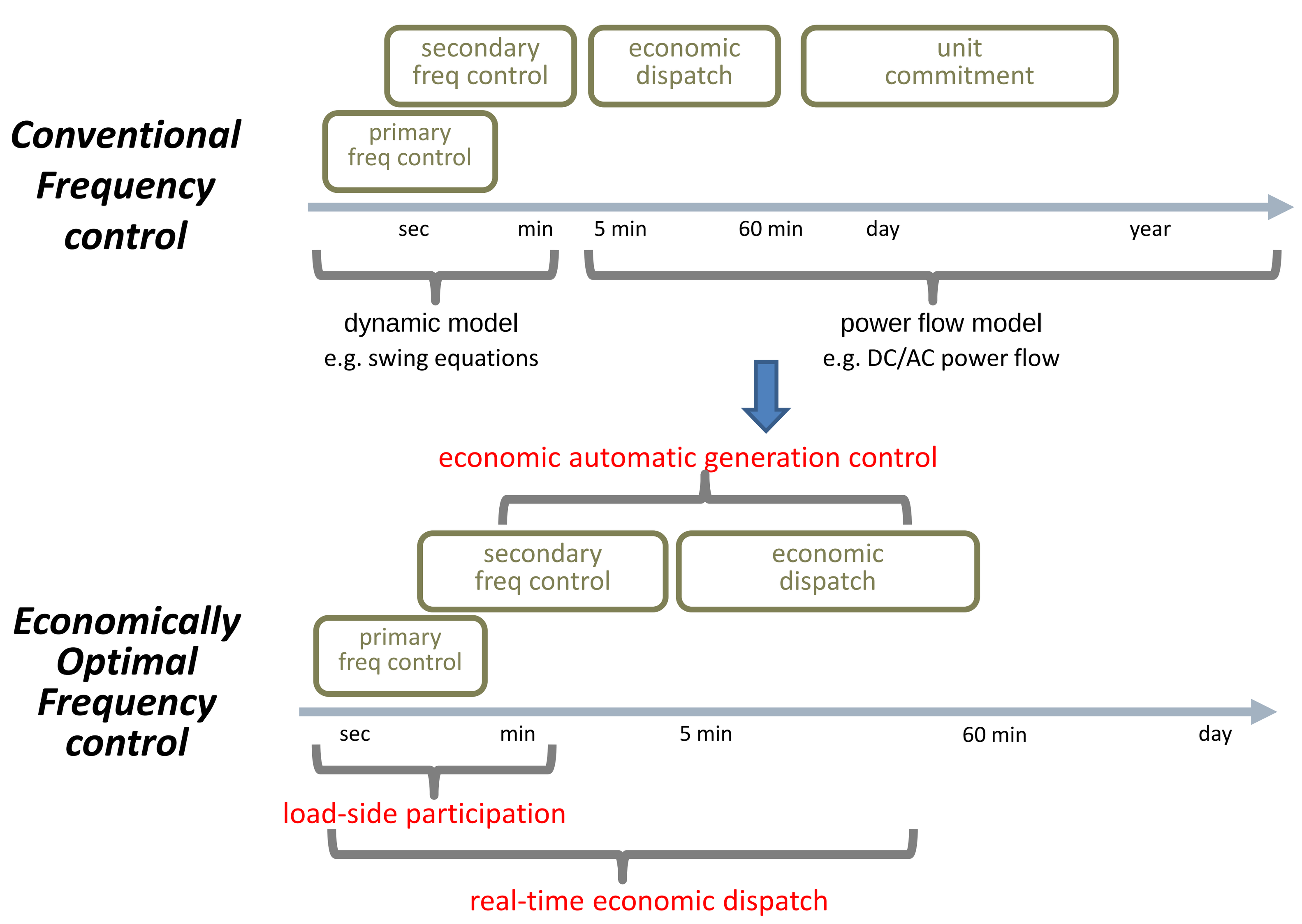
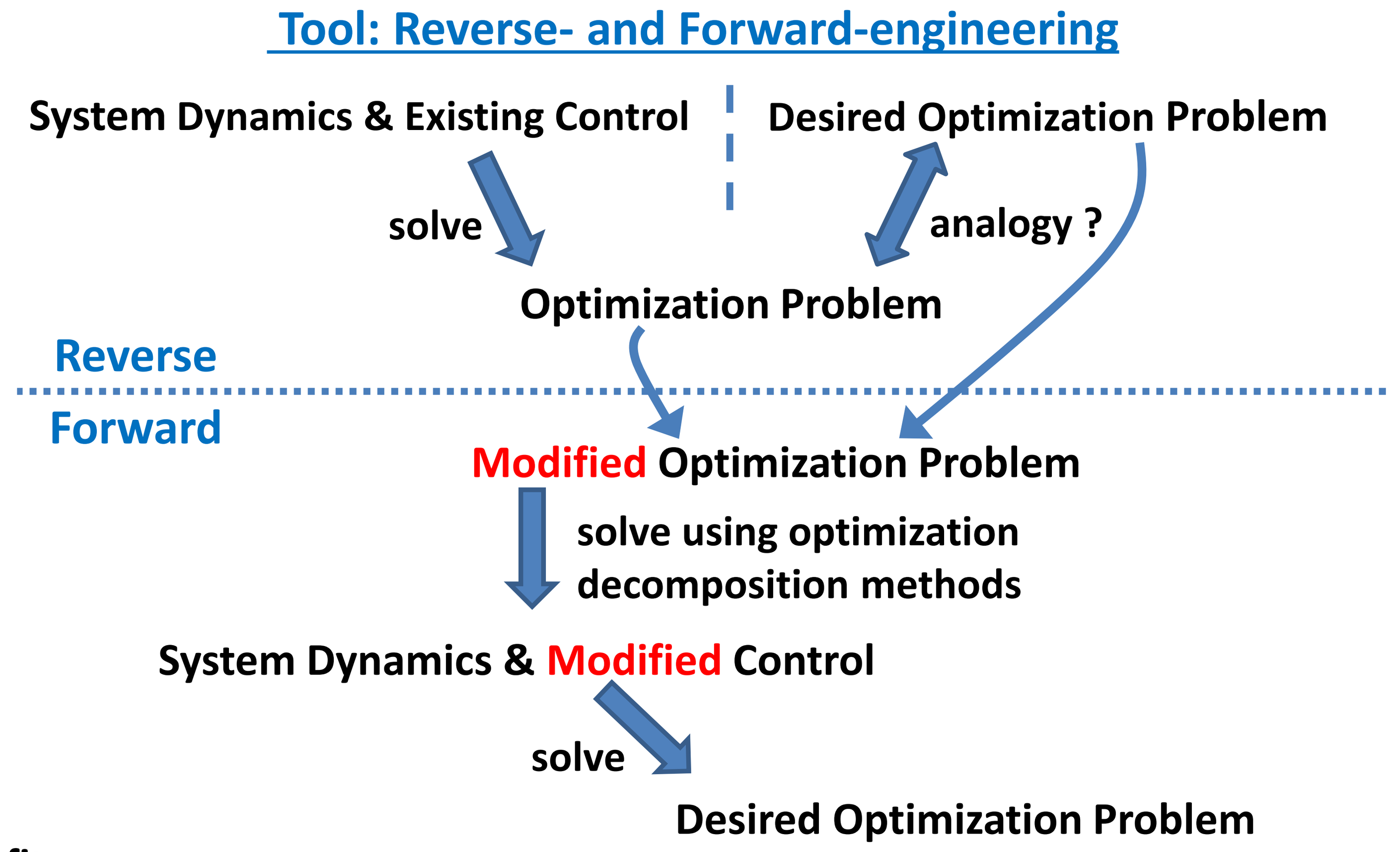


Goal: the closed-loop dynamical system can automatically track the optimal solution of an economic optimization problem (e.g. economic dispatch, optimal power flow) and the control scheme can be implemented in a distributed (i.e. communicating with neighbours) and closed-loop (i.e. no information of exogenous disturbances) manner.

Tool: Reverse- and Forward-engineering



Benefits:

(i) It allows us to embed different kinds of steady-state convex optimization problems;

(ii) If the desired optimization problem has a certain distributed structure, the resulting controller is distributed and can be implemented in a closed-loop manner;

(iii) the trajectories of the closed-loop system asymptotically converge to an equilibrium point at which the desired optimization problem is solved.

Main Results

I. Primary Load Frequency Control

System Dynamics

Generator bus: $M_i\dot{\omega}_i = -D_i\omega_i - P_{L_i} - d_i - \sum_{j:i\sim j} P_{ij}$

Load bus: $0 = -D_i\omega_i - P_{L_i} - d_i - \sum_{j:i\sim j} P_{ij}$

Real branch power flow: $\dot{P}_{ij} = b_{ij}(\omega_i - \omega_j)\ \ \forall\, i \to j$

Optimization Objective

$$\min \sum_i \left(c_i\left(P_{L_i}\right) + \frac{D_i}{2}\omega_i^2 \right)$$

over $P_{L_i} \in \left[\underline{P}_{L_i}, \overline{P}_{L_i}\right]$ and ω_i

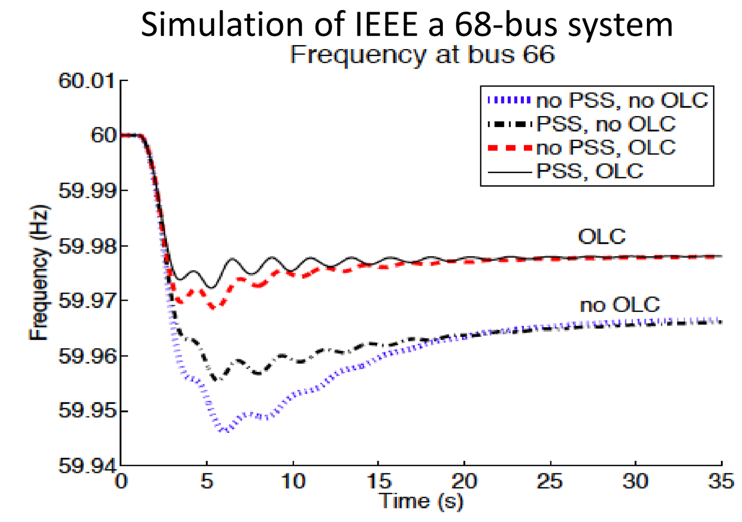
s. t. $\sum_i \left(P_{L_i} + D_i\omega_i\right) = -\sum_i d_i$

Load Control

$$P_{L_i} = \left[c_i'^{-1}(\omega_i)\right]_{\underline{P}_{L_i}}^{\overline{P}_{L_i}}$$

Swing dynamics plus the load control scheme serve as a distributed partial primal-dual gradient algorithm that solves the objective optimization problem. The load control scheme is completely decentralized.

Simulation of IEEE a 68-bus system



II. Economic Automatic Generation Control (AGC)

System Dynamics

Bus/area model: $M_i\dot{\omega}_i = -D_i\omega_i + P_{M_i} - d_i - \sum_{j:i\sim j} P_{ij}$

Real branch power flow: $\dot{P}_{ij} = b_{ij}(\omega_i - \omega_j)\ \ \forall\, i \to j$

Turbine-governor: $\dot{P}_{M_i} = -\frac{1}{T_i}(P_{M_i} - P_{C_i} + \frac{1}{R_i}\omega_i)$

ACE-based AGC: $\dot{P}_{C_i} = -K_i(B_i\omega_i + \sum_{j:i\sim j} P_{ij})$

Optimization Objective

$$\min_{P_{M_i}} \sum_i C_i\left(P_{M_i}\right)$$

s. t. $P_{M_i} = d_i + \sum_{j:i\sim j} P_{ij}, \forall i$

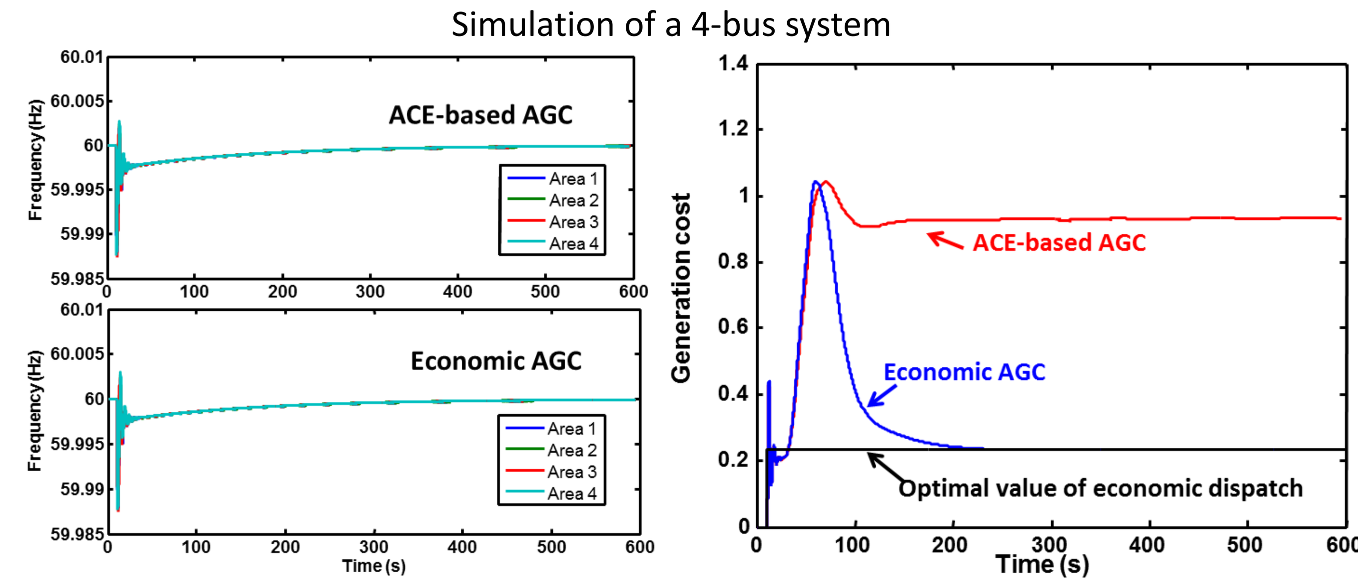
Economic AGC

$$\dot{P}_{M_i} = -\frac{1}{T_i}\left(\frac{1 - R_iK_iM_i}{R_i}C_i'(P_{M_i}) - P_{C_i} + \frac{1}{R_i}\omega_i\right)$$

$$\dot{P}_{C_i} = -K_i(D_i\omega_i + \sum_{j:i\sim j}(P_{ij} - \gamma_i + \gamma_j))$$

$$\dot{\gamma}_i = \varepsilon_i(M_i\omega_i - \frac{P_{C_i}}{K_i})$$

Simulation of a 4-bus system



Swing dynamics plus the ACE-based AGC serve as a partial primal-dual gradient algorithm to solve a convex optimization problem. This problem is then re-engineered to derive the economic AGC, which is distributed and easy to implement.

III. Distributed Control and Economic Optimality (general formulation)

System Dynamics

Network dynamics: $\dot{x}_i = \sum_{j:i\sim j} A_{ij}x_j + B_iu_i + C_iw_i$

Built-in controller: $\dot{u}_i = \sum_{j:i\sim j} D_{ij}x_j + \sum_{j:i\sim j} E_{ij}u_j + F_iw_i$

Optimization Objective

$$\min_{x_i,u_i} \sum_i f_i(x_i) + \sum_i g_i(u_i)$$

s. t. $\sum_{j:i\sim j} A_{ij}x_j + B_iu_i + C_iw_i = 0$

$\sum_{j:i\sim j} D_{ij}x_j + \sum_{j:i\sim j} E_{ij}u_j + F_iw_i = 0$

$h_i(x_i, u_i) \le 0$

"Economically Optimal" Control

$$\dot{u}_i = \sum_{j:i\sim j} D_{ij}x_j + \sum_{j:i\sim j} E_{ij}u_j + F_iw_i - \gamma P_{u_i}\Big(\frac{\partial g_i}{\partial u_i} - B_i^TK_{\varsigma_i}(\tilde{\varsigma}_i - x_i) - \sum_{j:i\sim j} E_{ji}^TK_{\lambda_j}(\tilde{\lambda}_j - u_j) + \mu_i\frac{\partial h_i}{\partial u_i} + K_{eu_i}(u_i - \hat{u}_i)\Big)$$

$$\dot{y}_i = -K_{y_i}\Big(\frac{\partial f_i}{\partial y_i} - \sum_{j:i\sim j} A_{ji}^TK_{\varsigma_j}(\tilde{\varsigma}_j - x_j) - \sum_{j:i\sim j} D_{ji}^TK_{\lambda_j}(\tilde{\lambda}_j - u_j) + \mu_i\frac{\partial h_i}{\partial y_i} + K_{ey_i}(y_i - \hat{y}_i)\Big)$$

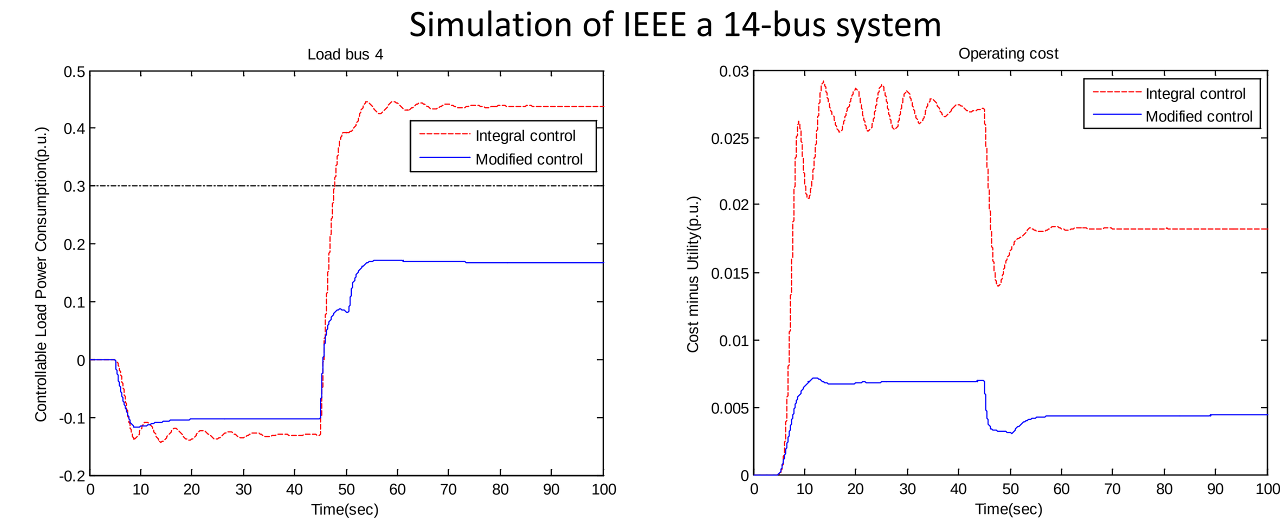
$$\dot{\tilde{\varsigma}}_i = \sum_{j:i\sim j} A_{ij}(x_j - y_j) - K_{e\varsigma_i}(K_{\varsigma_i}(\tilde{\varsigma}_i - x_i) - \hat{\varsigma}_i)$$

$$\dot{\tilde{\lambda}}_i = \sum_{j:i\sim j} D_{ij}(x_j - y_j) - \gamma P_{u_i}\Big(\frac{\partial g_i}{\partial u_i} - B_i^TK_{\varsigma_i}(\tilde{\varsigma}_i - x_i) - \sum_{j:i\sim j} E_{ji}^TK_{\lambda_j}(\tilde{\lambda}_j - u_j) + \mu_i\frac{\partial h_i}{\partial u_i} + K_{eu_i}(u_i - \hat{u}_i)\Big) - K_{e\lambda_i}(K_{\lambda_i}(\tilde{\lambda}_i - u_i) - \hat{\lambda}_i)$$

$$\dot{\mu}_i = k_{\mu_i}(h_i(x_i, u_i))^+_{\mu_i},\ \dot{\hat{u}}_i = \hat{K}_{eu_i}(u_i - \hat{u}_i),\ \dot{\hat{y}}_i = \hat{K}_{ey_i}(y_i - \hat{y}_i)$$

$$\dot{\hat{\varsigma}}_i = \hat{K}_{e\varsigma_i}(K_{\varsigma_i}(\tilde{\varsigma}_i - x_i) - \hat{\varsigma}_i),\ \dot{\hat{\lambda}}_i = \hat{K}_{e\lambda_i}(K_{\lambda_i}(\tilde{\lambda}_i - u_i) - \hat{\lambda}_i)$$

Simulation of IEEE a 14-bus system



System dynamics plus the built-in controller serve as a primal-dual gradient algorithm to solve a quadratic saddle point problem. This problem is then re-engineered to modify the controller, which converges to the optimal point of the desired optimization problem.

References:

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[2] N. Li, L. Chen, C. Zhao and S. H. Low, "Connecting Automatic Generation Control and Economic Dispatch from an optimization view ," in *Proc. of 2014 American Control Conference*, 2014, pp. 735-740.

[3] X. Zhang, N. Li, and A. Papachristodoulou, "Achieving real-time economic dispatch in power networks via a saddle point design approach," *Power and Energy Society General Meeting*, 2015, to appear.

[4] X. Zhang, N. Li, and A. Papachristodoulou, "Distributed optimal steady-state control using reverse- and forward-engineering," in *IEEE Conference on Decision and Control*, 2015, submitted.