Discrete-Time Monitoring of Power Grids

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***Abstract*—With high penetration of inverter-based renewable energy sources (IBRES) power system dynamics began to occur within shorter time frames. However, existing monitoring tools that rely on phasor measurements lack sufficient resolution to monitor such fast transients in the grid. To address this issue, this paper proposes a weighted least absolute value (WLAV) estimation-based monitoring tool that uses the same discrete samples of measured voltages and currents that are used by the phasor measurement units (PMU). Such samples are in general not made available to the users by the PMUs but instead they are processed for a full or fraction of a cycle to obtain the positive sequence phasors. Given that these discrete samples are readily available, there is little reason not to use them for tracking fast voltage and current transients introduced by inverter-based renewable energy sources. Performance of the proposed estimator is illustrated using a small IEEE 30-bus system modeled using appropriate discrete-time components and simulated measurements with Gaussian noise as well as gross errors.**

***Index Terms*—Least absolute value, discrete time modeling, state estimation, inverter-based resources.**

I. Introduction

Traditionally, SCADA measurements were commonly employed in power system monitoring tools. With the widespread use of PMUs, phasor measurements are started to be used in favor of SCADA measurements due to their higher resolution and lack of time skew. The higher resolution of measurements and linear relation between the system states and the phasor measurements enabled faster and more frequent estimation of system states by using linear state estimators [1]. In addition, synchronous machines and dynamic load models are attempted to be identified by dynamic state and parameter estimators using high-resolution phasor measurements [2]. Although these applications vastly improved the grid visibility, they are limited by the PMU resolution. However, the recent increase in the number of inverter-based resources (IBRs) connected to power grids necessitates even higher resolution measurements in order to capture their fast dynamics. The existing grid monitoring software tools cannot accomplish this.

Currently, IBR controls are formulated based on positive sequence network models and phasor measurements. However, there is growing evidence that such an approach falls short of accurately representing the operating conditions. Moreover, maintaining updated models of the installed equipment and frequently exchanging information with the system operators may not always be possible, creating biased or inaccurate results by network applications relying on this information. Even when IBR models are accurate, lack of model sharing with neighboring control areas may lead to undesired effects such as IBR controllers acting against each other [3]. These issues could be overcome with a monitoring tool that would enable grid-wide observability, which could ensure a more optimal operation of the grid. Instead of solely relying on component models and local measurements, a system-level monitoring tool may provide system-wide observability of the system status, even for sparsely measured areas.

The idea of estimating fast dynamics of power grids is not entirely new as evident from early work illustrating these ideas on a simple tutorial system using discrete-time measurement equations in anticipation of advancements in fast synchronized measurement technology [4]. That work is later extended in [5] using a weighted least squares (WLS) estimator and subsequently adding a post-estimation bad data identification stage via the largest normalized residual test [6]. In [7], WLS-based transient state estimation is proposed to identify the sources of disturbances in partially observable systems, where the system elements are modeled using state-space theory. Later, it is extended by incorporating discrete-time lumped models and modeling three-phase systems in [8] and [9], respectively. Also, in [10], the authors further extended their work with the use of lossy traveling wave transmission model for long transmission lines. These works generally addressed power quality monitoring instead of defining a generalized framework for grid-wide monitoring, and bad data identification is not considered or not thoroughly investigated. In [11], a numerical derivative-based transient state estimation is proposed, where only the Gaussian noise on the measurements is considered bad data.

Despite the fact that PMU technology has advanced significantly over the past two decades, most commercial PMUs do not provide access to their raw measurement samples. They provide either three-phase or simply the positive sequence phasors calculated by processing the raw input samples of voltages and currents. Yet, these raw ”point-on-wave” measurements can be very useful in capturing the fast dynamics of IBRs and

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constitute the main motivation of the presented work in this paper. It should be noted that the commonly used WLS-based state estimators described in the literature may not be able to meet the required cpu time requirements considering the very short time windows between measurement samples. This is primarily due to the iterative nature of post-estimation bad data identification processes. An alternative inherently robust estimator is needed to accomplish this fast monitoring task.

In this paper, a weighted least absolute value estimator-based monitoring tool that exploits point-on-wave measurements is proposed and developed. In order to comply with the resolution requirement of the point-on-wave measurements, the discrete models of the grid components are developed by using Bergeron's model. Thanks to the automatic bad data rejection capability of WLAV estimation, the proposed method does not require post-estimation bad data detection and identification process. Possible leverage measurements are identified by inspecting projection statistics, and the leverage measurements are weighted with the reciprocal of the projection statistics to reduce their effects on the estimation results. Note that the proposed monitoring tool is limited to balanced networks, as the grid components are represented with only single-phase Bergeron's discrete-time equivalent models. Also, the proposed monitoring tool assumes that the grid components are frequency independent. The extension of the incorporating frequency dependencies of the components is left as future work.

This paper is organized as follows: In Section II the problem formulation and the proposed method are given. In Section III, the simulation results are presented. Finally, the conclusion is given in Section IV.

II. Problem Formulation

A brief review of the models and associated assumptions used in formulating the state estimation problem will be given first for completeness. This will be followed by the formulation and numerical solution of the state estimation problem by using the weighted least absolute value (W-LAV)estimation approach. The motivation for introducing weights into the LAV formulation will also be explained in the sequel.

A. Discrete-time Modeling of Power System Elements

The network elements are required to be modeled with models that are compatible with sampling rates in the range of microseconds in order to comply with point-on-wave measurements. To achieve this, discrete-time models of elements are utilized, which are commonly used in transient modeling [12]. Each grid element is modeled with an equivalent impedance network that consists of equivalent impedances and history terms relating the past states to the present time. The short lines and shunt elements are modeled with their lumped equivalent models, while the long lines are modeled with the single-phase lossless model, the so called Bergeron model or the constant parameter (CP) model [12], [13]. Transformers, which are typically modeled using pi-models, are converted into discrete time short-line models. The discrete-time measurement equations associated with these models are briefly described next.

1) Short-line model of line $k-m$:

a) Measurement equations:

$$\begin{aligned} i_{km}(t) &= \frac{v_k(t)}{R_C} - \mathbf{I_{C,k}}(t-\Delta t) \\ &\quad + \frac{v_k(t) - v_m(t) + R_L \cdot \mathbf{I_L}(t-\Delta t)}{R_L + R_r} \\ i_{mk}(t) &= \frac{v_m(t)}{R_C} - \mathbf{I_{C,m}}(t-\Delta t) \\ &\quad + \frac{v_m(t) - v_k(t) - R_L \cdot \mathbf{I_L}(t-\Delta t)}{R_L + R_r} \end{aligned} \tag{1}$$

b) History terms:

$$\begin{aligned} \mathbf{I_L}(t-\Delta t) &= \mathbf{I_L}(t-2\Delta t) + \frac{2}{R_L} \cdot v_L(t-\Delta t) \\ \mathbf{I_{C,k}}(t-\Delta t) &= -\mathbf{I_{C,k}}(t-2\Delta t) + \frac{2}{R_C} \cdot v_k(t-\Delta t) \\ \mathbf{I_{C,m}}(t-\Delta t) &= -\mathbf{I_{C,m}}(t-2\Delta t) + \frac{2}{R_C} \cdot v_m(t-\Delta t) \end{aligned} \tag{2}$$

where i_{km} and i_{mk} are current flows between buses k and m. I_L, $I_{C,k}$, and $I_{C,m}$ denote the history terms related to the inductor and the capacitors respectively. R_L, R_C, and R_r are the equivalent resistances of the inductor, capacitor, and resistor in the equivalent impedance model of the discrete-time model. The simulation time and time step are denoted with t and Δt, respectively. The equivalent resistances R_L and R_C are calculated as $(\Delta t/2L_t)$ and $(2C_t/\Delta t)$ respectively, where L_t and C_t are the total line inductance and the total line capacitance. Hence, they remain constant as long as the Δt remains the same. Finally, v_k, v_m, and v_L are voltages of buses k, m, and the voltage between the terminals of the inductor.

2) Models of shunt elements connected at bus k to ground:

a) Measurement equations:

$$i^{sh}_{k,L}(t) = \frac{v_k(t)}{R_L} + \mathbf{I_L}(t-\Delta t) \tag{3}$$

$$i^{sh}_{k,C}(t) = \frac{v_k(t)}{R_C} - \mathbf{I_C}(t-\Delta t) \tag{4}$$

$$i^{sh}_{k,r}(t) = \frac{v_k(t)}{R_r} \tag{5}$$

b) History terms:

$$\mathbf{I_L}(t-\Delta t) = \mathbf{I_L}(t-2\Delta t) + \frac{2}{R_L} \cdot v_k(t-\Delta t) \tag{6}$$

$$\mathbf{I_C}(t-\Delta t) = -\mathbf{I_C}(t-2\Delta t) + \frac{2}{R_C} \cdot v_k(t-\Delta t) \tag{7}$$

where $i^{sh}_{k,L}$, $i^{sh}_{k,C}$ and $i^{sh}_{k,r}$ are the currents flowing along the shunt inductor, shunt capacitor, and shunt resistor at bus k to ground, respectively. Note that the history terms are calculated and updated in a similar way as described in (7).

3) Long-line model of line $k-m$:

a) Measurement equations:

$$\begin{aligned} i_{mk}(t) &= \mathbf{I}_m(t-\tau) - \frac{1}{Z_{c,mk}} \cdot v_m(t) \\ i_{km}(t) &= \mathbf{I}_k(t-\tau) + \frac{1}{Z_{c,km}} \cdot v_k(t) \end{aligned} \tag{8}$$

b) History terms:

$$\begin{aligned} \mathbf{I}_m(t-\tau) &= i_{km}(t-\tau) + \frac{1}{Z_c} \cdot v_k(t-\tau) \\ \mathbf{I}_k(t-\tau) &= i_{mk}(t-\tau) - \frac{1}{Z_c} \cdot v_m(t-\tau) \end{aligned} \tag{9}$$

The history term update equations in (9) can be rewritten in a recursive form as follows:

$$\begin{aligned} \mathbf{I}_m(t-\tau) &= \mathbf{I}_k(t-2\tau) + \frac{2}{Z_c} \cdot v_k(t-\tau) \\ \mathbf{I}_k(t-\tau) &= \mathbf{I}_m(t-2\tau) - \frac{2}{Z_c} \cdot v_m(t-\tau) \end{aligned} \tag{10}$$

where i_{mk} and i_{km} denote current flows between buses m and k, while I_m and I_k are history terms of current flows at bus m and bus k side, respectively. τ denotes the travel time between bus k and m, and Z_c is the characteristic impedance of the line. For a lossless line with length d, per length inductance L' and per length capacitance C', the traveling time τ and the characteristic impedance Z_c are calculated as follows:

$$\begin{aligned} \tau &= d/v = d \cdot \sqrt{L'C'} \\ Z_c &= \sqrt{\frac{L'}{C'}} \end{aligned} \tag{11}$$

The traveling speed of the wave is denoted by v and is constant for a lossless transmission line. Thus, τ is also constant.

B. Weighted Least Absolute Value (WLAV) State Estimation Formulation

When expressed in discrete-time measurements and bus voltages will be linearly related, thus the estimation solution will be non-iterative. Consider a system with n buses and m measurements. The system state vector $x(t)$ will contain the bus voltage samples at time instant t:

$$x_i(t) = v_i(t), \quad i = 1, 2, ..., n. \tag{12}$$

The samples i.e. "point-on-wave measurements" that are obtained from raw samples used by PMUs are the voltages, line currents, and bus current injections at time t. The measurements incident to buses k and m at time t, $z(t)$, will include the following:

$$z(t) = [v_k^T(t)\, v_m^T(t)\, i_{km}^T(t)\, i_{mk}^T(t)\, i_k^T(t)\, i_m^T(t)]^T \tag{13}$$

where $v_k(t)$ and $v_m(t)$ are voltages at buses k and m, $i_{km}(t)$ and i_{mk} are line currents between buses k and m, and $i_k(t)$ and $i_m(t)$ are current injections at time t.

The measurement and state relation for short lines, shunt elements, and for long lines at time t can be expressed in the following form:

$$\begin{aligned} z(t) = Hx(t) &+ h_{lumped}(t-\Delta t) \\ &+ h_{long}(t-\tau) + e \end{aligned} \tag{14}$$

where H is the jacobian, h_{lumped} and h_{long} are the vectors of history terms corresponding to lumped elements and long lines respectively, and e is the vector of measurement errors. Note that the jacobian H, is composed of only equivalent resistances in discrete-time models. Hence, it is constant and needs to be computed only once if Δt remains unchanged.

The history term vector, h, is calculated from the previous state estimation results. Hence, it is treated as a known quantity for subsequent time instants' estimation. Hence, the calculated history terms are subtracted from the measurements and the resulting corrected measurement vector z_h is obtained as follows:

$$\begin{aligned} z_h(t) = z(t) - h_{lumped}(t-\Delta t) - h_{long}(t-\tau) \\ z_h(t) = Hx(t) + e \end{aligned} \tag{15}$$

Use of the corrected measurement vector defined in (15) transforms the state estimation problem into the standard form similar to traditional state estimation problems, enabling the use of the same solution techniques. Note that by eliminating the history terms, all variables are now in the same time instant, t. Therefore, the time variable can be dropped from the equations in the rest of the discussion.

WLAV estimator aims to minimize the sum of absolute values of measurement residuals as given below:

$$\sum_{i=1}^{m} |r_i| = c^T |r| \tag{16}$$

where
$c^T = [w_1 \quad w_2 \quad \dots \quad w_m]$ is a $(1 \times m)$ vector of weights.
$r^T = [r_1 \quad r_2 \quad \dots \quad r_m]$
$r_i = z_{h,i}^m - \hat{z}_{h,i}$
r_i is the i^{th} measurement residual,
$z_{h,i}^m$ and $\hat{z}_{h,i}$ are the i^{th} measurement value and i^{th} measurement value calculated with estimated states, respectively.

In more compact form, measurement equations can be written as below:

$$\hat{z}_h = H\hat{x} + r \tag{17}$$

where
$\hat{z}_h^T = [\hat{z}_{h,1} \quad \hat{z}_{h,2} \quad \dots \quad \hat{z}_{h,m}]$
$\hat{x}^T = [\hat{x}_1 \quad \hat{x}_2 \quad \dots \quad \hat{x}_n]$
$\hat{x}$ is the estimated state vector.

Then, the WLAV estimation problem can be formulated as the following optimization problem:

$$\begin{aligned} \min \quad & c^T |\bar{r}| \\ \text{s.t.} \quad & \bar{z_h} - H\bar{x} = \bar{r} \end{aligned} \tag{18}$$

By rearranging the equations and introducing additional variables that are strictly non-negative, the WLAV optimization problem given in (18) can be expressed as an equivalent linear programming (LP) problem given below:

$$\begin{aligned} \min \quad & c^T y \\ \text{s.t.} \quad & My = b \\ & y \geq 0 \end{aligned}$$

$$\begin{aligned} c^T &= [Z_n \quad O_m] \\ y &= [X_a \quad X_b \quad U \quad V]^T \\ M &= [H \quad -H \quad I \quad -I] \\ b &= \hat{z}_h \end{aligned} \tag{19}$$

where Z_n is the $1 \times 2n$ vector consisting of zeros and O_m is the $1 \times 2m$ vector consisting of weights, w_i where;

$$w_i = min\{1, \frac{1}{PS_i}\}, \quad i = 1...m. \tag{20}$$

PS_i is the projection statistic of i^{th} measurement, which is further explained in the next section. X_a and X_b are $1 \times n$, and U and V are $1 \times m$ vectors where;

$$\begin{aligned} \bar{x} &= X_a^T - X_b^T \\ \bar{r} &= U^T - V^T \end{aligned} \tag{21}$$

The optimization problem defined in (19) can be solved efficiently by one of many LP solvers.

C. Bad Data and Leverage Measurements

Measurement equation (17) implicitly assumes that the history terms, that are calculated using the previous time instants' estimation results, do not contain any gross errors. This assumption may be violated when measurements carry gross errors impacting not only the state estimation results for that time instant but also subsequent time instants due to the propagation of errors via the history terms. Consecutive bad data in a few time instants may have significant impact on the accuracy of the estimation and could even lead to large oscillations in estimation results. Therefore, bad data detection and identification is a crucial feature of any newly proposed state estimator. Although the solution time for a WLS based estimator is generally shorter than its WLAV counterpart, this advantage rapidly disappears in the presence of multiple bad data. This is due to the fact that WLS estimator needs to execute an iterative post-estimation bad data detection and identification algorithm based on repeated application of the largest normalized residual test in order to detect, identify and remove measurements with gross errors one at a time. This process quickly becomes quite cpu intensive in case of multiple bad data. Considering the short time window between the two consecutive estimation instants, WLAV becomes the preferred option since unlike the WLS method, its solution time remains fairly insensitive to the number of gross errors [1] and it is inherently robust against multiple bad data given sufficient measurement redundancy.

It is important to discuss one of the common vulnerabilities of any estimator namely the so called leverage measurements. Such measurements are defined by their undue influence on the estimated states and therefore constitute threats to the accuracy and reliability of state estimators. Their residuals will be insignificant irrespective of their accuracy, i.e. if they carry gross errors they will go undetected. Measurement configuration, type and network topology all play a role in the manifestation of leverage measurements. Generally, injections at buses with large number of incident branches or branches with very different branch parameters act as leverage measurements. For a measurement set that is composed of voltage, current flow, and current injection measurements, current injections have typically the highest potential to be leverage measurements. In this paper, potential leverage measurements are identified by using the metric "projection statistics" of the measurement jacobian H as described in [14]. The calculated projection statistics of the identified leverage measurements can be used to down weight their influence. One way to accomplish this is incorporating weights in LAV estimator to yield a WLAV estimator. Note that the projection statistics remain unchanged unless the measurement Jacobian H or the assumed measurement error covariances change. Therefore the leverage measurement identification must be performed only once after the initialization. Also, note that the modeling of the branches may change the projection statistics, hence, the leverage measurements. The long-line model virtually decouples its terminal buses at a given time instant. As a result, having lines modeled as long-lines reduces the likelihood of having many leverage measurements compared to those represented by short-line models. This will be illustrated in the simulations section below.

III. Simulation Results

The proposed method is tested on a small IEEE 30-Bus system whose network diagram, measurement configuration and line types are shown in Fig. 1. Transformers are modeled using π circuits similar to short-lines. The measurement set includes voltage, line current flow, and bus current injections. Measurement errors with a standard deviation of 0.001 are added to the measurements using a Gaussian distribution to simulate the noise. The long lines are assumed to have identical τ for simplicity, and Δt is taken as 10% of the τ.

A. Leverage measurement identification

The leverage measurements are identified by inspecting the projection statistics of H. The measurements corresponding to higher projection statistics have a higher likelihood of being leverage measurements. Current injection measurements of buses with many connections and current flow measurements of lines that are weakly connected are naturally prone to being leverage measurements. However, the leverage likelihood of those measurements may be reduced by converting the relevant branch models from short-line to long-line where appropriate, to exploit their naturally decoupled model. Fig. 2 shows the projection statistics of two cases where the line $12-13$ is modeled first as a short-line and then as a long-line. The projection statistics of measurements v_{13} and $i_{12,13}$ are shown

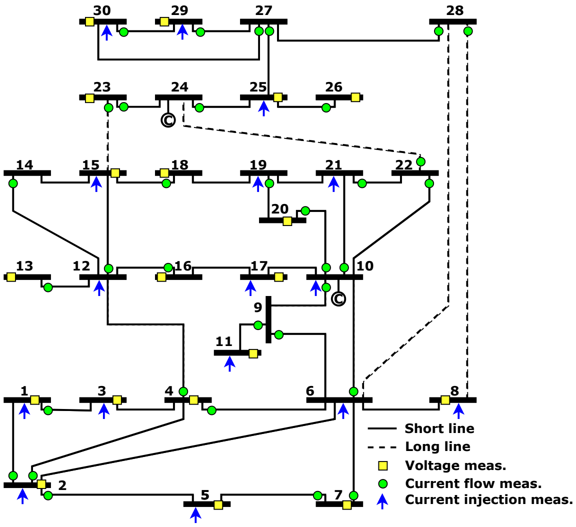

Fig. 1. IEEE 30-Bus test system.

to be significantly reduced, while the projection statistic of i_{12} is moderately decreased when long line model is used. This case demonstrates the impact of modeling on the likely appearance of leverage measurements.

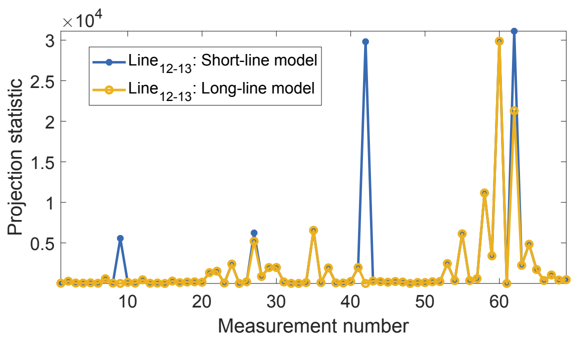

Fig. 2. Measurement set projection statistics.

B. Performance without gross errors

In this section, the performance of the WLAV, LAV, and WLS estimators are compared in the absence of bad data. Fig. 3 and 4 present the estimated voltage and current injections at bus 14. Note that the injection at bus 14 is not measured, yet it is estimated enabling the system operator to monitor the output of a source and/or load behind bus 14. As evident from these results, all three estimators perform quite satisfactorily with acceptable accuracy.

C. Performance under gross error

In this section, the estimators are compared under gross error. At $t = 0.795$ ms, a gross error is injected into the current injection measurement at bus 2 by reversing the polarity of the measurement and scaling its value by 10, i.e., $i_2^{bad} = -10 \times i_2^{measured}$. Fig. 5 and Fig. 6 present the estimated voltage and current injection results at bus 2, where Fig. 7

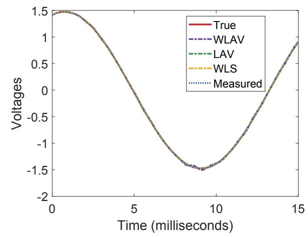

Fig. 3. Estimated voltages at bus 14, without bad data.

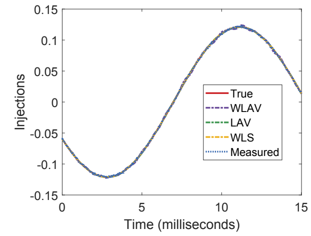

Fig. 4. Estimated current injections at bus 14, without bad data.

and Fig. 8 show the estimation results at bus 14, respectively. In the presence of a single gross error, WLS and LAV fail to provide an accurate estimation, where WLS is biased by the bad data and LAV is biased due to I_2 being a leverage measurement. Additionally, inaccurate estimation results affect the subsequent time instants' estimation results due to the propagation of errors in the history terms calculated using biased results. The effect of bad data remains in the system for quite sometime until it is filtered out by the estimators. On the other hand, WLAV handles the gross error by automatically rejecting it.

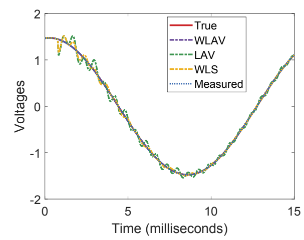

Fig. 5. Estimated voltages at bus 2, under gross error.

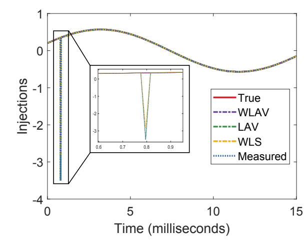

Fig. 6. Estimated current injections at bus 2, under gross error.

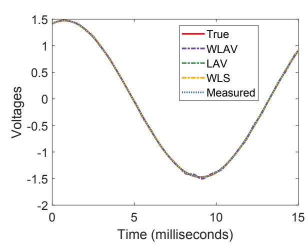

Fig. 7. Estimated voltages at bus 14, under gross error.

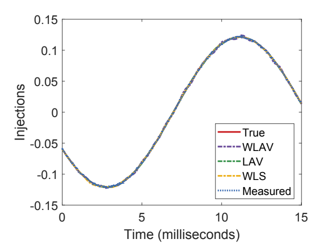

Fig. 8. Estimated current injections at bus 14, under gross error.

D. Loss of communication link

In this section, a failure in a communication link is simulated. The communication is lost at $t = 2.019$ ms, and the current injection measurement at bus 2 can therefore not be updated and remains at its recorded value, until $t = 9.158$ ms. Fig. 9 and Fig. 10 show the estimated voltage and current injection results at bus 2, where Fig. 11 and Fig. 12 present the results at bus 14, respectively. During the interval, the WLAV remains accurate while the WLS and LAV deviate from the true value. Furthermore, the WLS and LAV require additional time after the repair of the communication link before reaching an acceptable accuracy, due to the biased history terms.

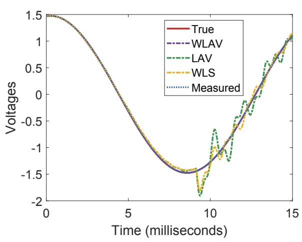

Fig. 9. Estimated voltages at bus 2, during loss of communication link.

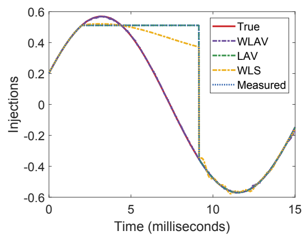

Fig. 10. Estimated current injections at bus 2, during loss of communication link.

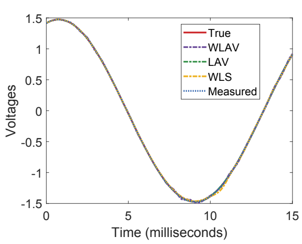

Fig. 11. Estimated voltages at bus 14, during loss of communication link.

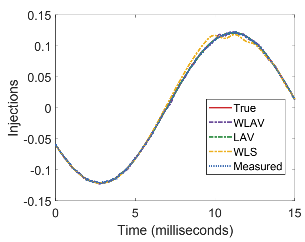

Fig. 12. Estimated current injections at bus 14, during loss of communication link.

IV. Conclusion

Existing phasor measurement-based estimators are inadequate to monitor the fast dynamics in the grid. This paper addresses this issue by developing a WLAV-based state estimator that utilizes point-on-wave measurements that are also used as inputs to PMUs. The proposed method is tested on IEEE 30-Bus system. Simulation results indicate that the proposed method is capable of accurately estimating the system states in the presence of bad data in the measurements. It is shown by simulations that WLAV remains robust against bad data and eliminates the need for a separate post-estimation bad data detection and identification process. Furthermore, the influence of any existing leverage measurements is properly reduced to avoid biased estimation results if such measurements carry gross errors.

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