A Directed Acyclic Graph Neural Network for AC Optimal Power Flow

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INTRODUCTION

Background: AC OPF is an NP-hard problem, and its solution can be time-consuming by traditional optimization techniques.

Motivation: To solve AC OPF more efficiently, a Direct Acyclic Graph Neural Network (DAG-NN) is proposed in this paper, which enables an explicit design of a neural network utilizing the intrinsic structural information of the problem to be solved.

APPROACH

- **Steps**

1) Formulating the process of solving AC OPF as a compositional function.
2) Representing the compositional function by a DAG.
3) Constructing a DAG-NN by realizing each node of the DAG by a shallow neural network.

- **How to use a DAG to represent a compositional function**

Considering a compositional function below:

$$f(x_1, x_2, x_3) = x_1 x_2 - x_2 x_3 + \cos x_3$$

The DAG of this compositional function can be used to construct an NN by replacing its each node by a shallow NN.

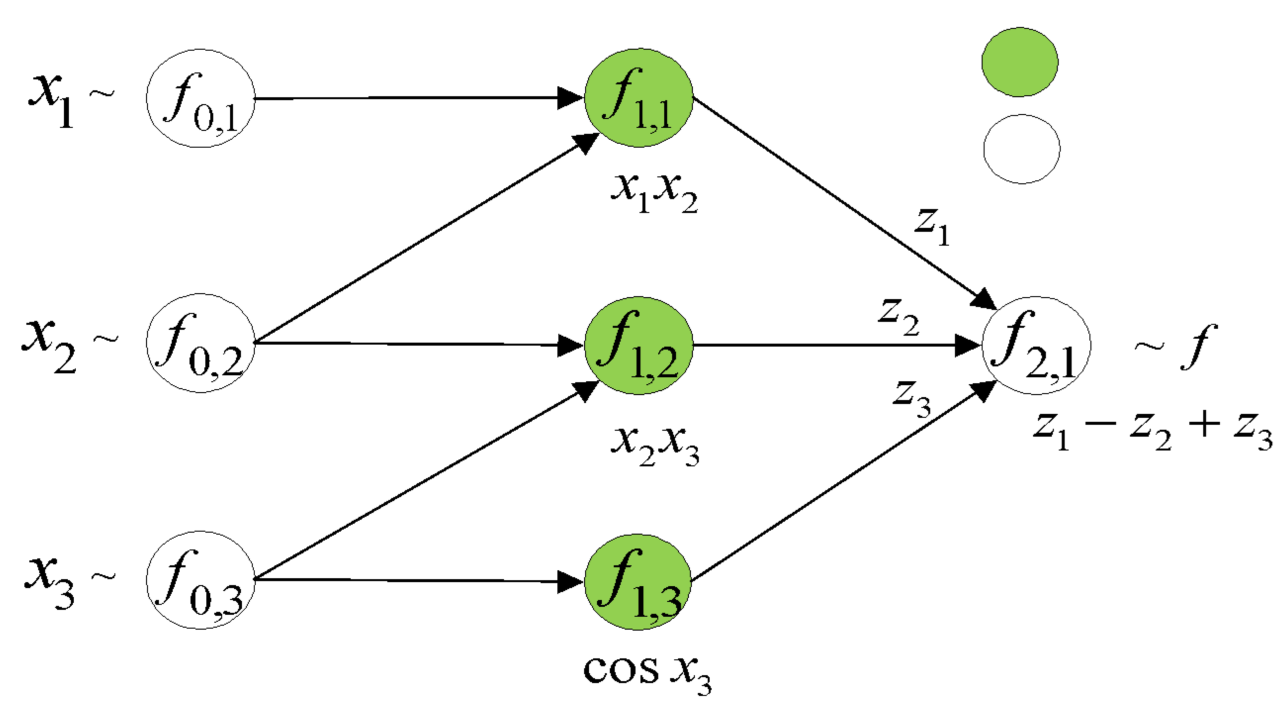

Fig.1 A DAG of the compositional function f

- **The DAG-NN for AC OPF**

Reformulates an iterative Newton-Raphson based AC OPF algorithm as a compositional function.

$$\mathbf{y} = [PG, QG, \delta, V, \lambda]$$

$$\mathbf{y}^{k+1} = \mathbf{y}^k + \Delta\mathbf{y}^k = \mathbf{y}^k + \mathbf{h}(\mathbf{y}^k)$$

$$N(\mathbf{y}) = \mathbf{y} + \mathbf{h}(\mathbf{y})$$

$$\mathbf{h}(\mathbf{y}) = -\left[\nabla^2 \mathrm{L}\right]^{-1} \times \nabla \mathrm{L}$$

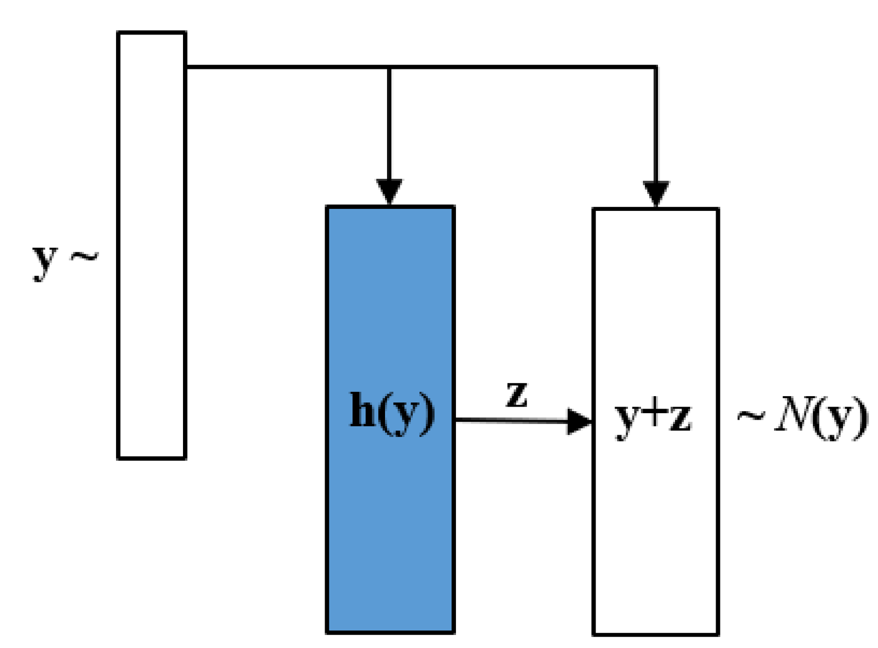

Fig.2 1-DAG of one Newton-Raphson iteration

- **Error analysis on the DAG-NN**

The error of the DAG-NN for AC OPF can be obtained from the following:

$$\left\|\phi(\mathbf{y}) - \phi^{NN}(\mathbf{y})\right\|_p \le \frac{L^K - 1}{L - 1} \sum_j L_j^{\mathbf{h}} \varepsilon_j + e_N$$

CASE STUDIES

- **2-bus system**

TABLE I. MEAN ABSOLUTE ERRORS OF THE COST FUNCTION FOR THE 2-BUS SYSTEM

Case		Top 1	Top 10	Top 50
DAG-NNs	Case 1: 2-DAG-NN	1.08E-07	1.47E-06	2.18E-06
	Case 2: 3-DAG-NN	1.01E-08	9.20E-08	1.48E-06
	Case 3: 4-DAG-NN	7.59E-09	3.23E-08	3.21E-07
Traditional NNs	Case 4: 2 hidden layers	2.63E-06	5.58E-06	2.74E-05
	Case 5: 3 hidden layers	2.70E-06	3.45E-06	6.87E-06
	Case 6: 4 hidden layers	1.33E-06	2.50E-06	5.31E-06

All DAG-NNs have smaller mean absolute errors than traditional NNs with the same NN size.

- **PJM 5-bus system**

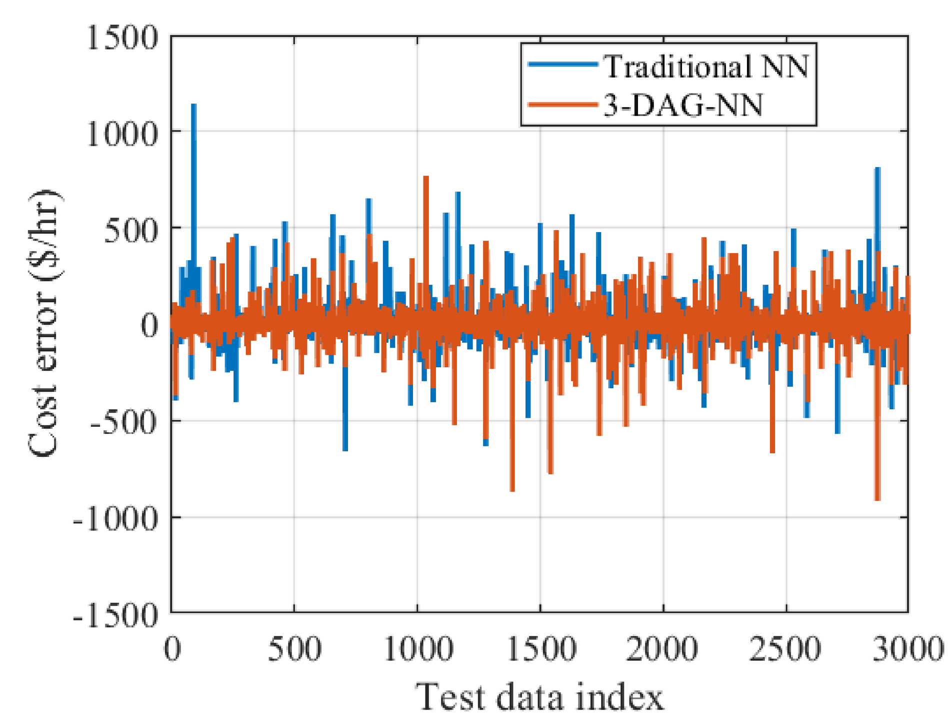

Fig.3 The error of the cost function for different NNs

Starting from the warm start points obtained by the DAG-NN and the traditional NN, all testing cases can converge to accurate results, whose relative errors of the costs are within [-0.0005%, 0.0005%].

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