A Study on Energy Preservability of Runge-Kutta Methods in Power System Simulation

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INTRODUCTION

- **Background:** (**a**) Conventional Runge-Kutta methods can not preserve the total energy of the simulated system. (**b**) Damping estimation can be introduced errors.
- **Contribution:** This paper studies explicit formulae on how the total energy of the simulated system trajectory can change with the integration time step.
  - By using the Hamiltonian system formulation of a single-machine-infinite-bus system, the existence of a critical time step for energy-preserving simulation is discovered.
  - The formulae are used to evaluate the error in observed damping as well as the correction if the simulation is conducted for an extended period with a time step different from the critical time step.
- **Advantages**: (**a**) An explainable instruction of power system simulation time step can be given. (**b**)A relative large time step can be used to evaluate damping.

DERIVATION

Given a Hamiltonian System

$$\dot{p} = -Kq \qquad \dot{q} = p / M$$

Energy can be calculated by

$$E = p^2 / 2M + Kq^2 / 2$$

Adopting R-K4 approach, iteration process is

$$k_1 = h \times f(x_N) \qquad k_2 = h \times f(x_N + 0.5k_1)$$

$$k_3 = h \times f(x_N + 0.5k_2) \qquad k_4 = h \times f(x_N + k_3)$$

$$x_{N+1} = x_N + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

Total energy at time t can be obtained

$$E_t = (1 - h^6 K^3 (\frac{-Kh^2 + 8M}{576M^4}))^{t/h} E_0$$

The R-K 4 method for $K > 0$ and $M > 0$ has a critical time step h_c by which simulation can preserve the total energy

$$h_c = \sqrt{\frac{8M}{K}}$$

Assume $h < h_c$ for harmonic oscillator with damping. **The numerical damping, which is the portion of fake damping due to numerical simulation, is given by**

$$\zeta_e = -\frac{\ln D}{2h} \times \sqrt{\frac{M}{K}} \times 100\%.$$

$$D = (1 - h^6 K^3 (\frac{-Kh^2 + 8M}{576M^4}))$$

CASE STUDIES

Step size(s)	Damping Ratio of SMIB System: Prony Analysis (%)	Error ε_1 of Prony Analysis (%)	Numerical Damping (%)	Error ε_2 due to numerical damping (%)
0.1	4.24	0.01	0.0015	0.0117
0.2	4.28	0.03	0.0469	0.0149
0.3	4.52	0.27	0.3372	0.0637
0.4	5.45	1.20	1.3179	0.1131
0.5	7.89	3.63	3.6864	0.0484
0.6	13.05	8.80	8.4067	0.3967
0.7	22.59	18.34	16.7444	1.5943
0.8	35.50	31.25	31.7761	0.5213

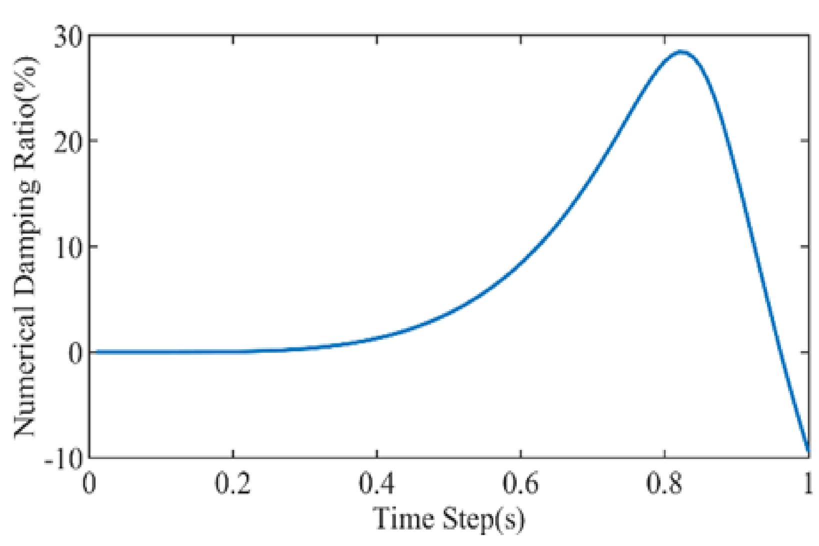

Fig. 1. For SMIB system

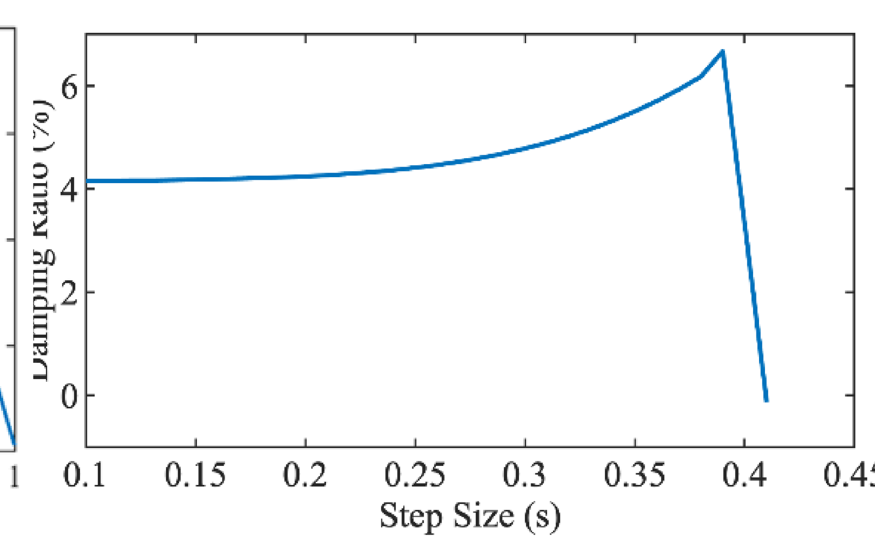

Fig. 2. For two area system

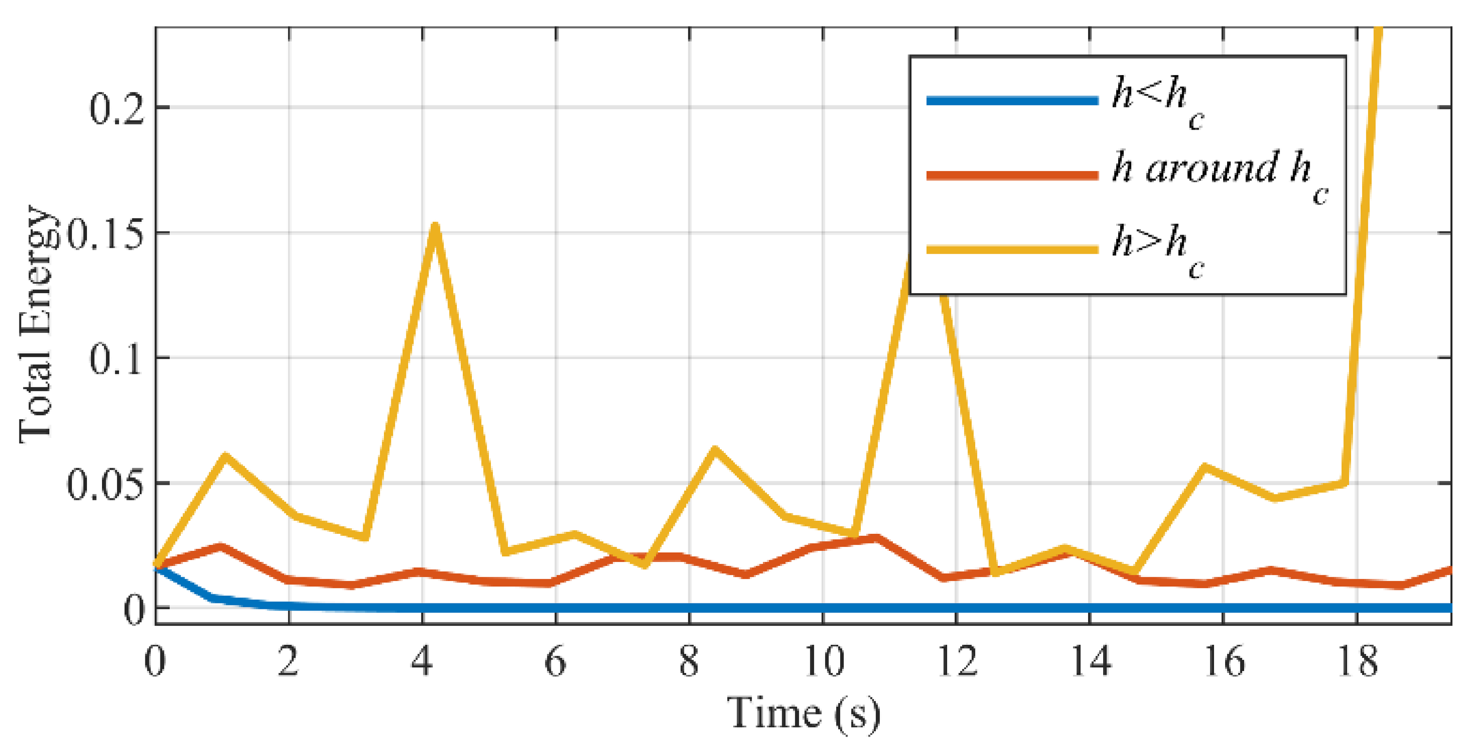

Fig. 3. Total energy for the SMIB system

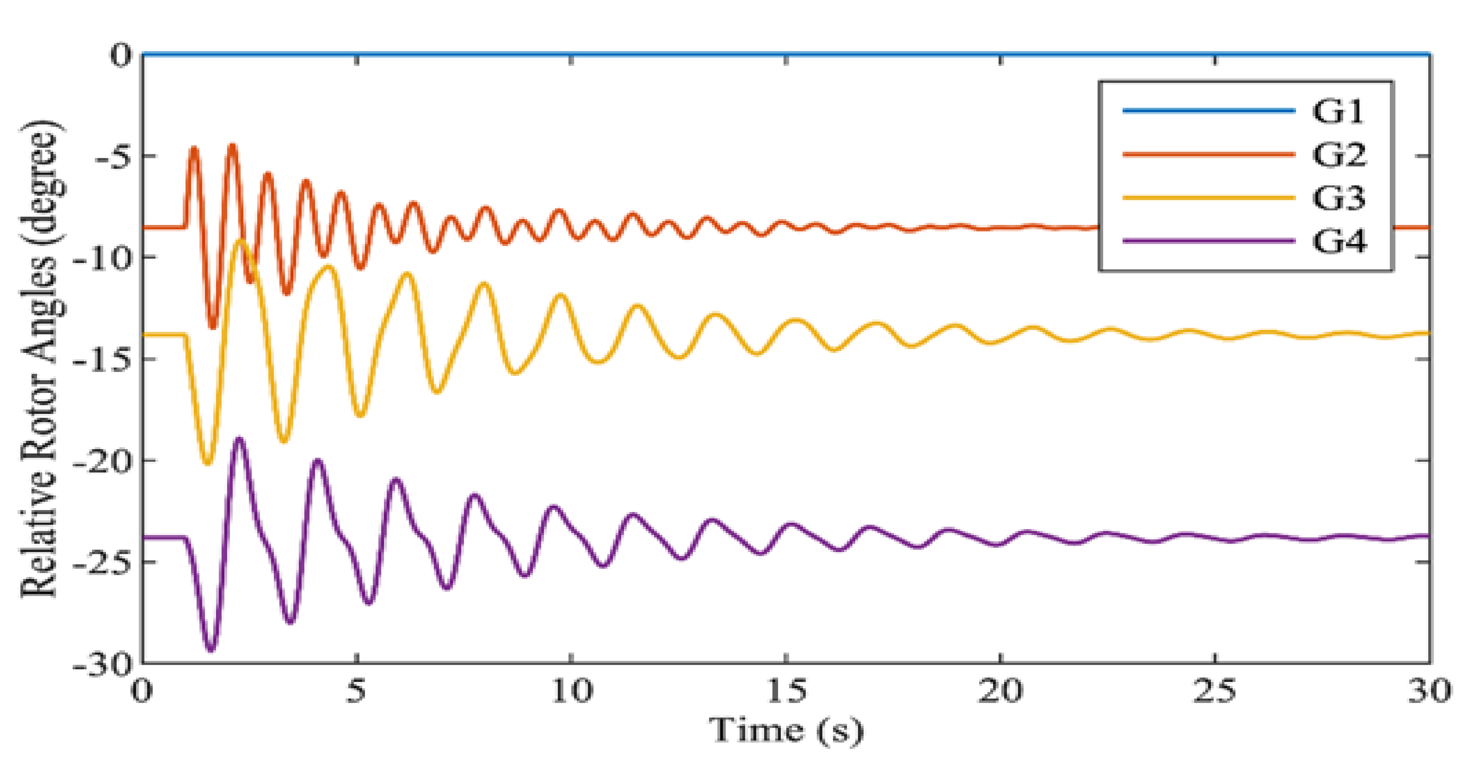

Fig. 4. Relative rotor angles

CONCLUSION

(a) The concept of numerical damping has been adopted to evaluate the fake portion of damping told from simulation results if energy is not preserved. **(b)** The derived formulae and conclusions have been validated on the SMIB system and also tested on a two-area system.